A

COMPLETE SYSTEM

OF

LAND SURVEYING,

BY THE

CHAIN SIMPLY.

Illustrated with Twenty-two Copperplates.

A

SYSTEM

OF

LAND SURVEYING,

WITH THE CHAIN SIMPLY:

WHEREIN IS GIVEN THE NECESSARY

PROBLEMS AND THEOREMS OF GEOMETRY,

TOGETHER WITH

A COMPREHENSIVE

Table of Logarithms;

WITH NUMEROUS

PRACTICAL EXAMPLES

OF

GEOMETRY UPON GROUND, MEASURING, PLANNING, AND CALCULATING THE SUPERFICIAL TENT, FROM A SINGLE FIELD TO THE LARGEST ESTATE:

ALSO, CONTAINING

An Entire New Method of finding the Products and Sums of Products of Numbers, superior to that of Logarithms.

BY PETER FLEMING,

CIVIL ENGINEER AND LAND SURVEYOR.

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ADVERTISEMENT.

IT may be proper to remark, that this PART is intended as a complete treatise of LAND SURVEYING with the CHAIN only; while it forms the elements, and contains preliminary instructions to be observed, when perusing the succeeding Parts to be published.

The object of having proposed this system of LAND SURVEYING AND LEVELLING, is to form an English Treatise, which in systematic order treats of the Theory and Practice of this branch of Mathematical Science, to that extent to which it now should be understood and practised; for although there are several books upon Land Surveying, these are either very much abridged, or filled with methods which should now be obsolete, when we consider the present perfection of Surveying and Drawing Instruments: besides the practice of many Measurers rather show the want of the application of Mathematical knowledge, in a profession which is purely Geometrical.

In the First Section, I have given those PROBLEMS and THEOREMS, the greater part of which I consider absolutely necessary for the practice and knowledge of a Land Surveyor; but of the first, I have not given the demonstrations; because in doing this alone would have almost constituted a volume of itself, without those advantages which will be found in the study of Euclid's Elements, or other similar treatises which are already published.—The THEOREMS, with their Co-

rollaries, are demonstrated in a manner more to familiarize the learner to the measure of angles upon the circle, and thereby answering to the figure of those instruments, which he will have much occasion to use in practice, than containing all that Mathematical definition, and strictness of demonstration, which is to be found in the Geometrical works referred to; but these will be found some of the most useful in their applications.

The matter composing the Second Section, contains chiefly the rules and applications of LOGARITHMS to Arithmetical Calculations, with appropriate Examples. In the comprehensive Table which is annexed, will be found an improvement in the arrangement of the PROPORTIONAL PARTS, as explained in the text, by which, with very little trouble, or merely inspection, this Table becomes the Logarithms from 1 to 100,000.

The Third Section is titled LAND SURVEYING, because wholly relating to the method and practice of Measuring Land, and Planning therefrom, as done with the Chain only. Among the first articles is Practical Geometry upon the ground, exemplified by several useful Problems.—The description of the Drawing Instruments which follow, are merely those necessary for protracting Plans from chain dimensions; and afterwards to the conclusion, there are exemplified all the cases of measuring and protracting Single Fields, Farms, Roads, &c.; but which is altogether confined to those methods only affording strict verifications: so that rude methods, as by the general use of the Cross Staff, are only noticed as such.

The Fourth or last Section is wholly devoted to SUPERFICIAL EXTENT or AREAS, and begins with the particular demonstrations and

Rules for finding the area of the Square, Rectangle, Triangle, Rhomboid, Trapezoid, Polygon, Circle, Segment, and Sector of Circles, and Ellipse; in illustration of all of which I have given calculated examples, both by Natural and Logarithm numbers. The finding of the areas of Single Fields, either when measured together, or separately, is exemplified in the same manner, and here I have demonstrated and applied the ratio of Triangles having one common Angle; by which is obtained a method for calculating the area of each field, without measuring them separately. Also is given the different and best methods for calculating the area from a Delineation or Plan, with calculated examples of each, as used in practice. There is likewise added several Tables, which will be found very useful to the practising Land Surveyor.

In the latter part of this Section is given a method, which in its application I presume is NEW, for finding the product of any two numbers, from a Table of Square numbers, without multiplication; it will also find the sum of the product of any number of pairs of multipliers by the same means, whereby it is more peculiarly applicable to find the area, either severally or collectively, from any number of dimensions which are of Triangles and Rhomboids, or all other figures of which the area is the result of multiplication. This method avoids the constant liability of error in many actual multiplications of natural numbers, for Addition and Subtraction are the only operations necessary; and is much more expeditious than Logarithms, by avoiding the necessity of finding the product of each pair of multipliers, before the whole sum when only desired can be known.

Throughout the whole of this Part, I have endeavoured to explain the Theory of Land Surveying with the Chain only, without refe[illegible]

to other Geometrical works, but only to the Problems and Theorems in this, by which the learner may gain a competent and correct knowledge of both Theory and Practice within the same volume. It will also be observed, besides the Problems and Theorems under Practical Geometry, others are given where found to be particularly applicable, as in Articles 48 and 50, by which the several uses as applied to Surveying, are directly shewn in Practical Examples, and which will be continued throughout the succeeding Sections to be published.

P. FLEMING.

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SYSTEM

OF

LAND SURVEYING.

PRACTICAL GEOMETRY.

GEOMETRY is the Science which discovers the relations and properties of that representation of Magnitude and Extension, which we name figure. The demonstrations of Geometry being derived from the most simple and natural conceptions, are such as afford the clearest evidence to the most intricate propositions, whence it becomes a boundary between truth and uncertainty, in all objects to which it is applicable. The applications of Geometry are so various, that there is almost no Art or Science but of which it is either the basis, or can be made subservient to discover its elements, and if any new effect be discovered, this generally will develope the principle or cause. Astronomy, Navigation, and Geography we know are wholly dependent upon Geometry; and Architecture, Sculpture and Painting, have their rules from its demonstrations: also, in constructing the maps of countries, fortresses and canals, both the Military and Civil Engineer require Geometry; and Land Surveying and Levelling, as constituting the first and necessary operations of the last professions, are its most direct applications.

PLATE I.

Definitions.

1. A *point* is that which marks position, but not magnitude.

2. A *line* is length without breadth.

3. *Lines* are either straight or curved.

4. A *straight* or *right line*, lies all in the same direction, between its extreme points, as A.

5. A *curve line* is that of which not any two adjoining portions of it together, is a straight line, as B.

6. Lines are said to be *parallel*, when all the distances between their opposite or corresponding parts keep the same, however far they may be produced, as C.

7. *Oblique lines* are those in which all the distances between their corresponding parts keep not the same when produced, as D.

8. An *angle* is formed by two lines divergent from the same point, as A B and A C, which is usually expressed the angle B A C, as Fig. 1.

9. A *right angle* is that which is equal to either of the two adjacent angles made equal to each other, by a straight line meeting another between its extremities, as D A B, or B A C, Fig. 2.

10. An *acute angle* is a divergence less than that of a right angle, as E A C, Fig. 2.

11. An *obtuse angle* is a divergence greater than that of a right angle, as D A E, Fig. 2.

12. A *perpendicular* is a line meeting another and making a divergence with that line equal to a right angle, as A B is perpendicular to A C, Fig. 2.

13. A *triangle* is a plane surface contained by three straight lines; and has its names from the relations of its sides and angles; for if

all the sides are equal, it is called *equilateral*, as Fig. 3; if two [illegible] *isoceles*, as Fig. 4; and if all unequal is *scalene*, as Fig. 5; also, if containing a right angle is *right angled*, as Fig. 6; or an obtuse angle is *obtuse angled*, as Fig. 4; and if all acute is *acute angled*, as Fig. 3.

14. Any side of a triangle may be called the *base*, and the angular point opposite, is the *vertex*; but of a right angled triangle, the side opposite the right angle is the *hypothenuse*.

15. The angle at the vertex is called the *vertical angle*.

16. A *quadrilateral* figure is contained by four straight lines; but is denominated from the relation of its sides and angles; for if the sides and angles are equal, it is a *square*, as Fig. 7; if the opposite sides are equal, and all the angles equal, it is a *rectangle*, as Fig. 8; if the sides are equal, and the opposite angles only equal, it is a *rhombus*, as Fig. 9; if opposite sides and angles are only equal, a *rhomboid*, as Fig. 10; if two of its sides are parallel, and the other two equal, it is a *trapezium*, as Fig. 11; and if two of the sides are parallel, but all unequal, it is a *trapezoid*, as Fig. 12.

PLATE II.

17. A *diagonal* of a quadrilateral figure, is a straight line, which joins the opposite angular points, as A B, Fig. 13.

18. Plane figures having more than four sides, are named *polygons*, and have their names from the number of their sides or angles; as, a polygon of five sides is a *pentagon*, a *hexagon* six, a *heptagon* seven, an *octagon* eight, &c.; and a polygon is said to be *regular*, if its sides are all equal, but if unequal, *irregular*.

19. The boundary of any right lined figure is called the *perimeter*.

20. A *circle* is a plane figure, contained within a curved line, which

is called the *circumference*, every point in which is equally distant from a certain point within, named the *centre*, as Fig. 14.

21. The *radius* of a circle is the distance from the centre to the circumference, as C D.

22. The *diameter* of a circle is equal to twice the radius, or a straight line passing through the centre, and terminating on both sides by the circumference, as A B.

23. An *arc* of a circle is any portion of the circumference, as D E B.

24. A *chord* is a straight line joining the extremities of an arc, as D B.

25. A *segment* is that part of a circle which is bounded by an arc and its chord, such as D E B, and D B.

26. The half of the circle is called a *semicircle*, as F A G, and the fourth part a *quadrant*, as A C G.

27. A *sector* is any part of the circle bounded by an arc, and radii joining the extremities of that arc and the centre, as D E B C.

28. Every arc is the *measure* of the angle or divergence of the radii which joins its extremities to the centre, by being compared to the whole circumference, as D E B is of the angle D C B.

29. The *altitude* of a figure is a perpendicular falling upon the base, or on it produced, from the remotest point opposite.

30. Figures are said to be *equal*, when their corresponding parts coincide, and *equivalent*, when they contain the same measure.

31. Lines are said to *intersect* when crossing each other.

PROBLEMS.

PROBLEM I.

The three sides D E F, *being given to construct a triangle.*

Draw A B equal D, and upon the centre A with the distance E, describe a circle. Describe another circle from the centre B, with the distance F meeting the former in C.—Draw A C and B C, and A B C is the triangle.

PROBLEM II.

To bisect a given angle E C F.

At equal distances A and B from the angular point C as centres, describe two arcs of the same radius, intersecting at D.—Draw C D and the angle is divided into two equal angles.

PROBLEM III.

To make one angle equal to another.

UPON A, the given angular point, with any radius describe an arc, cutting the two sides in B and C: with the same radius describe another arc, E F, from the point D, and from any point E, with the distance B C, intersect it in F.—Draw D E and D F, and the angle F D E is equal to the angle B A C.

PROBLEM IV.

To draw a perpendicular to a given point in a straight line.

FROM the given point A of the straight line, make B and C equally distant, and upon these points, as centres, describe arcs of the same radius, intersecting each other at D.—Then draw D A, which is a perpendicular to B C.

PROBLEM V.

To draw a perpendicular from one extremity of a line.

TAKE any point C for a centre opposite the line, and upon the same side to which the perpendicular is to be drawn, and describe with the radius C B the circle A B D, and draw the diameter A C D.—Join D and B, and D B will be the perpendicular required.

PROBLEM VI.

To draw a perpendicular from a given point without a straight line.

Upon the given point A as a centre, with any radius cut the straight line in B and C, and from the points B and C, with equal radius make an intersection D.—Draw A D till it meet the straight line in E, and A E is a perpendicular to B C.

PROBLEM VII.

To bisect a straight line.

From A and B, the extremities of the straight line, describe arcs making intersections with each other in C and D.—Draw C D, and its intersection with A B, divides the straight line into two equal parts.

PROBLEM VIII.

Through a given point C, *to draw a line parallel to a given straight line* A B.

From any point D, as a centre, in the line A B, describe the arc C E, and from C, with the same radius describe the arc D E.—Make D F equal to E C, and through C and F draw C F, which is the parallel required.

PROBLEM IX.

Upon a straight line A B, *to construct a square.*

Upon A and B, with the radius A B, describe the arcs A C and B D, intersecting at E, and bisect B E in F.—Make E D and E C each equal to E F, and join A D, D C, and B C, which will be a complete square.

PROBLEM X.

To divide a straight line A B, *into any number of equal parts.*

From the extremity A of the given line A B, draw A C at any acute angle, and B D parallel to A C.—Repeat upon A C and B D, from A and B, any convenient distance, the number of times the division is required: join A C and B D, by drawing lines between the opposite and corresponding points, and the same number of intersections will be made upon A B, equally distant from each other.

PROBLEM XI.

Two right lines A B *and* B C *being given, to find a mean proportional.*

Join A B and B C in one straight line, and bisect it in D; describe the semicircle A E C, and erect the perpendicular B E, which is a mean proportional to A B and B C, or A B : B E : : B E : B C.

PROBLEM XII.

To find a third proportional to two given lines A B *and* B C.

Upon the extremity of A B draw B C perpendicular; also draw the hypothenuse A C, and bisect it in D, with the perpendicular D E: then upon E, with the distance A, describe the arc A C F, and produce A B to F.—B F is a third proportional to the lines A B and B C; or, A B : B C : : B C : B F.

PROBLEM XIII.

To find a fourth proportional to three given lines a b, b c, and a d.

Make A B equal to the first, and A D equal to the third, and from B, the extremity of the first, draw B C equal to the second, at any convenient angle to A B; also, through the point C, draw A C produced to E.—Draw D E parallel to B C, meeting A C in E, and D E is the fourth proportional; or, A B : B C : : A D : D E, and A C : C B : : A E : E D.

PROBLEM XIV.

The side of a polygon being given, to describe the polygon to any number of sides whatever.

Upon one extreme of the given side A B, describe a semicircle of any radius, and divide it into the same number of equal parts, as the sides of the required polygon, for instance five. Then draw lines from the centre through the points of division, but omitting the two last; and with the distance of the side A B, from A or B intersect each successively from the next.—Join these intersections, which will complete the polygon.

PROBLEM XV.

On a given diagonal to describe a square.

Bisect the given diagonal A B, by the perpendicular D E, and upon C, the point of bisection, with the distance A or B, describe the circle A E B D.—Join A E, E B, B D, and D A, and the square is complete.

PROBLEM XVI.

To inscribe a square in a given triangle.

Draw the perpendicular C D, and make B E perpendicular and equal to A B; also join E D, and draw F G parallel, and G H and F I perpendicular to A B.—H G F I will be the inscribed square.

PROBLEM XVII.

To bisect the arc of a circle A C B.

DRAW the chord A B, and bisect it in the point D, by a perpendicular produced to C, and the arc A C is equal to the arc B C.

PROBLEM XVIII.

Given an isoceles triangle A B C, *to construct another on the same base but with half the verticle angle.*

BISECT A B in the point E; join E C, which produce till C D be equal to C A or C B, and draw A D and D B.—A D B is the isoceles triangle required.

PROBLEM XIX.

Given an arc A D B, *to find the centre, and complete the circle.*

DRAW the chord of the given arc A B, and bisect it by the perpendicular D C; join A D, and from A draw A C, making an angle D A C equal to A D C.—The intersection C is the centre of the circle required.

PROBLEM XX.

To draw a tangent B D, *to a given circle through a given point* A.

From the given point A, draw the radius A C, and perpendicular to A C draw B D through the point A.—The straight line B A D is the tangent.

PROBLEM XXI.

A tangent line B D *being given, to find the point* A *where it touches the circle.*

Take any point E on the tangent B D, and from E to the centre draw E C: bisect E C in F, and with the radius F C or F E describe the semicircle C A E, cutting the tangent and semicircle in A, which is the point required.

PROBLEM XXII.

Through any three points A B C, *to describe the circumference of a circle.*

Join the three given points A B and C, and draw perpendiculars bisecting the lines A B and B C, produced till they meet in the point D, which is the centre of the circle required.

PROBLEM XXIII.

In a given circle A B D, *to describe three equal circles, which shall touch one another, and also the periphery of the given circle.*

From the centre C, bisect the circle by the right lines C A, C B, and C D: join A D, and produce C D till D G be equal to the half of A D; draw A G, and, parallel to it, D E meeting C A in E: make B F and D H each equal to A E, and upon E, F, and H, as centres, describe the circles through A, B, and D, which will touch one another.

In the same manner may any number of equal circles be made to touch one another within a given circle, by first dividing its circumference into the same number of equal parts as that of the circles required.

PROBLEM XXIV.

On a given straight line A B, *to describe the segment of a circle which shall contain a given angle* C.

Draw A D, making an angle B A D equal to C, erect A E perpendicular to A D, make E F to bisect A B at right angles and meeting A E in E, and from this point as a centre, and with the distance E A, describe the required segment A G B.

If the angle be a right one, the segment is a semicircle described upon A B.

PROBLEM XXV.

Three points being given A B C, *to find a fourth* P *to which, if lines be drawn from the three former, shall be in the ratio of three given lines respectively.*

JOIN the three given points, and make A F equal to *a*, and A I equal to *c*; also make the angle A F G and A I K equal, each, to A C B, and from the centres F and G, with the radii *b* and A K, respectively, describe two arcs intersecting in H: then draw H F and H A, and draw B P to make the angle A B P equal to the angle A H F.—Produce A H, meeting B P in P, which is the point required.

PROBLEM XXVI.

To describe a triangle A B C, *similar to a given one*, A N M, *so that three lines may be drawn from its angular points to another point, which shall be equal to three given lines respectively.*

LET A K, A F, and A D be the given lines: draw D E and K G, making the angles A D E and A K G, each equal to the given angle A N M, and intersecting A N in E and G. From D and E as centres, with the distance A F and A G, describe two arcs intersecting in H: draw A H and make A P equal to A D, and from the point P, with the distances A F and A K, respectively cut A M and A N in the points B and C.—Join P B and P C, and A B C is the triangle required.

PROBLEM XXVII.

Given the diagonals B E *and* A D, *and two opposite sides* A B *and* D E, *and the angle made by the inclination of the given sides to each other, to construct a trapezium.*

MAKE A B upon any line A C equal to one of the given sides, and make the angle C B G equal to the given angle: draw B G equal to the other given side, and upon A and G, as centres, with the respective lengths of the given diagonals, describe arcs intersecting each other at D.—Make D E equal and parallel to G B, and join D B and E A, and A B D E will be the trapezium required.

PROBLEM XXVIII.

To draw from a given point A *in the circumference of a given circle* A H D, *a given straight line through the circle, so that it shall be cut in any given ratio by the circumference.*

DRAW the given line A B through any point H of the circle, and divide it in I, to the given ratio; also draw indefinitely another line, A E, at any angle to A B. Produce A B, so as to make H C to A H, and also D E to A D, respectively, in the ratio of B I to I A: then, through the points C, E, and A, describe the circle C E A, and upon A, with the distance A B, cut it in the point L.—Join A L, which is equal to A B, and is cut in the given ratio by the circumference of the given circle A H D E in the point K.

ANOTHER METHOD.

FIND the centre M of the given circle, and from A, through M, draw A O, indefinitely cutting the circumference in N: make O N to N A, as B I is to I A, and bisect O A in the point P, which is the centre of the circle C E A.

PROBLEM XXIX.

Given two concentric circles, B H C *and* I G K, *and the ratio of the two parts of a straight line, the one intercepted between a given point* A, *in the innermost circle and its circumference, and the other part between the circumference of both the circles, to find the line and its position.*

DRAW B C through the given point A and the centre of the concentric circles, and divide A B in D, and A C in E, so that A E is to E C, and A D is to D B. in the same ratio of the parts of the straight line; then bisect D E in the point F, and upon F as a centre, with the distance F D or F E, cut the circumference of the inner circle in G, and through G draw A H, which is the line and its position.

PROBLEM XXX.

Given three concentric circles H I G, C L F, *and* B K E, *and the ratio of the two extreme segments of a straight line, the one intercepted be-*

tween a given point A in the innermost circle and its circumference, and the other between the second and outermost circle, to find the straight line and its position, and the ratio of the middle portion which lies between the first and second circumference.

Draw O P through the given point A and the centre of the circles, and make C D to A B, and F M to A E, in the same ratio of the given segments; then bisect D M in the point N, and with the distance N D or N M, cut the circumference of the outer circle in I.—Draw A I, and make L Q to A K, as C D is to A B; or also in the ratio of the segments: again, with the distance N Q, cut the circle H I G in T.—Join A T, which is the line, and S H the middle segment extremely near.

PROBLEMS

USEFUL FOR DRAWING THE PARTS OF *LARGE FIGURES.*

PROBLEM XXXI.

Three points A, B, *and* C *of an arc being given, to find a fourth* D *without these points, which likewise shall be in the circumference of the circle of that arc.*

From A, through the points B and C, draw indefinitely the lines A B and A C, and with any radius describe the arc E G: then make F G equal to E F, and through G draw A G towards D.—Upon C, with the distance C B, cut A G in D, which is the point required.

PROBLEM XXXII.

Three points of an arc A, B, *and* C *being given, to find a number of points lying between them, which shall be in that same arc.*

From A, through the points B and C, draw A B and A F, and with any radius describe an arc cutting A B in D, and A C in F: bisect the arc D F in the point E, and through E draw A H.—Join C B and bisect it in G, and draw G H perpendicular to B C, meeting A E in H, which is in the circumference with A B C. In the same manner may other points be found between B H and H C.

PROBLEM XXXIII.

To draw a tangent G B *to any arc* A B C *of a circle through any point* B, *without using the centre.*

From B towards C, make B D and D E equal, and draw the chord B E: with the radius B D describe the arc G F, and make D G equal to F D.—Through B draw G B, the tangent required.

PROBLEM XXXIV.

The arc A B C *being given, and a tangent* A D *to that arc, to find the point of contact, without using the centre.*

From any point H, without the arc, and with any radius, cut the arc in the points E and F, and join E F: bisect E F in the point G, and draw G H cutting the arc in B.—Make B I parallel to E F, meeting A D in I, and upon I as a centre, with the radius I B, describe the arc B K A, cutting the tangent and the given arc in A, which is the point of contact.

PROBLEM XXXV.

To draw an arc of any radius, the chord and height only being given.

Make A B the length of the chord, and D C the height of the arc required: draw C B, which will be the chord of half the arc, and perpendicular to C B draw B E; also make B F perpendicular to A B: then divide D B, C E, and B F, each into any but the same number of equal parts, as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Begin at 1 on B F, with the direction of 1 C, 2 C, 3 C, &c. and cut the lines 1 1, 2 2, 3 3, &c. in *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, which points will be in the circumference of the circle, and B *a*, *b c*, &c. C is half of the arc. By doing the same on the other half of the chord A D, the whole arc A C B may be drawn.

PROBLEM XXXVI.

Given the radius of a circle, to draw any portion of its circumference D I, *without using the centre.*

With the radius C A, any known part, such as one half, a third, or a fourth of the given radius of the required arc, describe the circle A B, and draw, indefinitely, any number of lines A D, A E, A F, A G, A H, and A I, cutting the circle A B in the points *a*, *b*, *c*, *d*, *e*, *f*, *g*: also make *a* D, *b* E, *c* F, &c. each respectively equal to as many times of *a* A, *b* A, *c* A, as the diameter A B is in the given radius of the arc: that is, if A B is made equal to half the radius of the given arc, *a* D is to be equal twice *a* A, *b* E twice *b* A, &c.: then through the points D, E, F, G, H, I, draw the arc required.

PROBLEM XXXVII.

Given three points A, B, *and* C, *to draw a line from any one of them towards the centre of the arc which the position of these points describe.*

Join A B, A C, and B C, and draw C D perpendicular to B C: make the angle B A E equal to the angle A C D, and A E is the line which will pass through the centre of the arc. But if the line is to be drawn from B, produce A B and A C to any distance G and H, and make the angle G B F equal to the angle H C D, and B F is the line required.

PROBLEM XXXVIII.

A point B *and a straight line* A C *being given in position, to describe a circle which shall pass through that point and the extremity* A *of the given line, but whose centre shall be upon that line.*

Join A and B, and bisect A B in the point D, by the perpendicular D E meeting A C in E: upon E as a centre, with the distance E A or E B, describe the circle A B F, which is that required.

PROBLEM XXXIX.

Two points A *and* B, *and a straight line* C D *being given in position, to draw a circle which shall pass through the given points, but whose centre shall be upon the given line.*

Join A and B, and bisect A B in E, by the perpendicular E F, meeting C D in F.—Upon F as a centre, with the distance F A or F B, describe the circle A B G.

PROBLEM XL.

Three points A, B, *and* C *of an arc being given, to complete the arc without using the centre.*

From any of the given points, as A, draw A D in the direction of the centre, and from any point E in A D, draw E B or E C: divide E A and E C or E B, each into any number of equal parts, for instance, three; then through any of the corresponding divisions, as 1, 1, describe a circle, having its centre in the line A D, and from E draw any number of lines, as E *f*, E *g*, E *h*, &c. cutting the circle in *a*, *a*, *a*, &c. and passing between the points A, B, C.—Make *a b*, *a c*, *a d*, *a e*, *a f*, &c. each equal to the number of times of *a* E, as E 1 is in 1 C or 1 A, and draw a line through *b*, *c*, *a*, *e*, *f*, *g*, &c. which is the arc required.

PROBLEM XLI.

To find the length of any arc A C B *of a circle.*

Draw indefinitely the chord A B, and bisect the arc A C B at C, and join A C.—Make A B equal to twice A C, and D E equal to one third of B D: then will A E be equal to the length of the arc A C B.

PROBLEM XLII.

Three points A B C *being given, lying in the circumference of a circle, and a fourth point* D, *to find another* F, *so that a circle passing through* F D, *and any given point, as* B, *of the given circumference, shall only touch it at that point.*

Draw B E, bearing upon the centre of the circle, and from D draw D F perpendicular to B E.—Make G F equal to D G, and F is the point required.

PROBLEM XLIII.

Three points A, B, *and* C *of a circle being given, and a fourth point* D *within or without that circle, to find other two points which, with the fourth, will describe a circle concentric to the three given points.*

Draw from any two of the three given points, as A and C, the lines A F and C E, tending to the centre, and upon any point G of the line A F, with the distance G D, describe the circle D F I: also upon H any point in C E, with the distance H D from the centre H, describe the circle D K E: then from D draw D I perpendicular to A F, and D K perpendicular to C E, meeting the respective circles in I and K, which are the two points required.

PROBLEM XLIV.

Two lines A B *and* C D, *bearing upon the same distant point, and also a point* E *in one of them being given, to find two other points, one of which is to be in the other* A B, *and both lying in the circumference of a circle passing through* E, *and whose centre is the point of meeting of* A B *and* C D.

Draw E F perpendicular to A B, and make G F equal to E G,

and F is the point required without A B: from F to any point H in C D, draw F H, which bisect in I; also draw I K parallel to F E, and bisect it in L, and from L draw L D perpendicular to I K, meeting C D in D: again, from F to any point M in A B, draw F M, which bisect in N; and from N, with the distance D I or D K, cut A B in B; upon this point as a centre, with the same radius N B, describe the arc N O, cutting A B in O.—Make O P equal to M O, and P is the other point which is in the line A B as required.

PROBLEM XLV.

Three points A, B, *and* C *being given, to draw from a given fourth point* D, *a line bearing upon the centre of the arc, described by the given position of three points.*

Draw lines A E and C F from any two of the points A and C, tending to the centre, and from A to any point G in C F, draw A G; bisect A G in H, and G C in I, and join H and I: also bisect H I in the point K, and draw K F perpendicular to H I, meeting C F in F: then join A D and C D, and bisect A D in L, and D C in M, and upon L and M as centres, with the distance F H or F I, describe arcs intersecting each other in N.—Through N draw D N, which is the line required.

PROBLEM XLVI.

The longest and shortest diameters, viz. A B *and* C D *being given, to*

describe the whole or any portion of an ellipse.

MAKE A F and B G each equal to C E or D E, and upon any of the points 1, 2, 3, 4, &c. in E F; with the distance F E from each respectively, cut D E in 1, 2, 3, 4, &c.: through 1 1, 2 2, 3 3, 4 4, &c. draw indefinitely 1 *a*, 2 *b*, 3 *c*, 4 *d*, and make 1 *a*, 2 *b*, 3 *c*, 4 *d* each equal to A F: then draw the curved line A *a b c d* C, which is a portion of the ellipse required.—In the same manner may the curve be described through C B D A, which will complete the ellipse.

GEOMETRICAL THEOREMS.

AXIOMS.

1. EQUAL arcs of the same radius have equal chords; so that if the arc A B C is equal to the arc D E F, the chord A C is equal to the chord D F.

2. Parallel lines, however, drawn through a circle, intercept equal arcs upon the circumference; as the arc A B is equal to the arc C D.

3. The greater chord in a semicircle subtends the greater arc; as if the chord A B is greater than the chord D E, the arc A C B is greater than the arc D C E.

4. Equal arcs subtend equal angles at the centre in circles of the same radius; as if the arc A B is equal to the arc D E, the angle A C B is equal to the angle D C E.

THEOREM I.

If a line C D *meet another line* A B, *the angles* A D C *and* C D B *are together equal to two right angles.*

DESCRIBE from D the semicircle A E C B, and draw E D perpendicular to A B: the arc A C is the measure of the angle A D C, and the arc C B is the measure of the angle C D B: (Def. 28.) also the arcs A E and E B, respectively, are the measures of the right angles A D E and E D B: (Def. 9.) but the arcs A C and C B are together equal to the arcs A E and E B, and consequently the angles A D C and C D E are together equal to two right angles.

Corollary.—Hence the whole circumference of a circle described from the point D of intersecting lines, being double the semicircle A E B, is the measure of four right angles.

THEOREM II.

If two right lines A B *and* C D *intersect each other, the opposite angles* C E B *and* A E D *are equal.*

Upon E as a centre describe the circle A C B D: the angles A E D and B E D are together equal to two right angles, (Theor. 1.) and C E B and B E D are also together equal to two right angles; therefore the angles A E D and B E D together are equal to the angles C E B and B E D, and consequently the angle A E D must be equal to the angle C E B.—In the same manner it may be demonstrated, that the angle A E C must be equal to the angle D E B.

THEOREM III

If the right line G F *cut other two parallel lines* A B *and* C D, *the outward angle* G E B *is equal to the opposite and inward angle* E F K.

Let E G be equal to E F, and draw a third line L M through G parallel to A B, and upon F and E as centres, with the distance F E or E G, describe the arcs E K and G B: because F E is equal to E G, and is parallel to E B, and G F a straight line, the intercepted arcs B G and E K are equal; (Ax. 2.) consequently the angles G E B and E F K are also equal. (Ax. 4.)

Cor.—The alternate angles A E F and E F K are equal, for A E F is equal to G E B. (Theor. 2.)

THEOREM IV.

The angle A B C *and* B A C *at the base of an isoceles triangle are equal.*

Upon C, with the distance C A or C B, describe the circle A D E B, and through C draw D E parallel to A B: the angle A C D is equal to the angle B A C, and the angle A B C is equal to the angle B C E: (Cor. Theor. 3.) but the arc A D is equal to the arc B E, for D E is parallel to A B; (Ax. 2.) therefore the angles A C D and B C E are equal, (Ax. 4.) and the angle A B C is equal to the angle B A C.

THEOREM V.

The angle C A B *at the centre of a circle, is double the angle* C D B *at the circumference, standing upon the same arc* B C *of that circumference.*

Through A draw D F, and draw D E parallel to A B, the angle A B D is equal to B D E, (Cor. Theor. 3.) and because the angle A B D is equal to A D B, (Theor. 4.) the angle B D E is equal to A D B; but the angle F A B is equal to the angle A D E, (Theor 3.) consequently the angle F A B is double the angle A D B.—In the same manner it may be demonstrated—the angle F A C is double the angle F D C; now taking the angle F A C from the angle F A B, and the angle F D C from the angle F D B, the remaining angle C A B at the centre is double the remaining angle C D B at the circumference. N. B. In applying this demonstration to Fig. 2, instead of taking from, add the angle F A C to the angle F A B, and the angle F D C to the angle F D B, the angle C A B at the centre is double the angle C D B, &c.

Cor. 1.—Hence the angle at the circumference standing on the diameter, or a semicircle, is a right angle; but if standing upon less than a

semicircle, is acute, and if upon greater, is obtuse, for this angle is measured by the half of the subtending arc of the circumscribing circle.

Cor. 2.—Angles at the circumference in the same segment, or standing upon the same arc are equal, for the half of the same arc is the measure of each.

THEOREM VI.

The three angles A B C, B C A, *and* C A B *of a triangle are equal to two right angles.*

Describe the circumscribing circle, and join the angular points A, B, and C to the centre D: the angle B D C is double the angle B A C, the angle B D A is double the angle B C A, and the angle A D C is double the angle A B C; (Theor. 5.) but the arcs measuring the angles B D A, B D C, and A D C are together equal to the whole circumference, or four right angles: (Cor. Theor. 1.) consequently the angles B A C, B C A, and A B C together are equal to half of the circumference, or two right angles.

Cor. 1.—Hence triangles having equal sides must be equiangular, and in every respect equal, for the half of the arcs intercepted by the corresponding sides upon the same circumscribing circle, are equal. (Ax. 4.)

Cor. 2.—Triangles which have two of their sides and the included angle equal, are equal to one another; because the arcs subtending the given angle upon the same circumscribing circle, are each equal to double the measure of that angle, and consequently the chords or third sides must be likewise equal. (Ax. 1.)

Cor. 3.—Also, equiangular triangles having one corresponding side

equal, have the other sides equal, for the arcs measuring the corresponding angles are equal upon circles of the same radius. (Ax. 1.)

THEOREM VII.

In every right lined triangle A B C, *the greater angle* A B C *is opposite the greater side* A C

DESCRIBE the circumscribing circle A B C, and join the angular points A, B, and C to the centre D: the angle A D C is double the angle A B C, the angle A D B is double the angle A C B, and the angle B D C is double the angle B A C; (Theor. 5.) but the arc A C is the measure of the angle A D C, and is greater than either of the arcs A B or B C, for A B C is the greater angle; (Theor. 5. Cor. 1.) therefore the chord A C is greater than either of the chords A B or B C, (Ax. 3.) and the angle A B C being measured by the half of A D C, is consequently greater than either of the angles B A C or B C A, and is opposite the greater side A C.

THEOREM VIII.

Parallelograms A C D B, E C D F, *and* E G H F, *standing upon the same base* C D, *or on equal bases* C D, G H, *and between the same parallels, are equivalent.*

BECAUSE C A is parallel to D B, and G E parallel to H F; A B,

E F, and C D are equal: also A E is equal to B F, therefore the triangles A C E and B D F are equivalent: (Theor. 6. Cor. 2.) from each of the triangles A C E and B D F take the triangle B E I, and the remaining trapeziums A B I C and E I F D are also equivalent. To each of the trapeziums A B I C and E I F D add the triangle C I D, then the parallelogram A B C D is equivalent to the parallelogram E B D G, and standing upon the same base C D.

THEOREM IX.

A triangle A B C *is the half of a parallelogram* A B C D, *when they stand upon the same base* A B, *and are between the same parallels* A B *and* C D.

For A C is equal to B D, and A B to C D, and C B is common to the triangles A B C and C D B, which are therefore equal: but the parallelogram A B C D is equivalent to the parallelogram A B C D: (Theor. 8.) consequently the triangle A B C is equal to the half of the parallelogram A B C D.

Cor. 1.—Hence every parallelogram is besected by its diagonal.

Cor. 2.—Triangles upon the same base, or on equal bases, and of the same altitude, are equivalent.

THEOREM X.

The square C A E D *described upon the hypothenuse of a right angled*

triangle, is equivalent to the squares A I H B *and* C F G B *of the two sides.*

Upon the sides C D and A E describe the triangles C F D and A I E, similar and equivalent to A B C, by making C F and I E each equal to B C, and F D and A I each equal to A B: produce A B and F D until they meet in G, and also C B meeting I E in H; also draw E K parallel to A I E, and make B M equal to I H, and L M parallel to A B: then C G is a square described upon C B, for the angles D F C, F C B, and C B G are right angles, and C F is equal to C B; also A H is a square upon A B, for the angles H I A, I A B, and A B H are right angles, and A I is equal to A B. (Def. 16.) The polygon A L M O is equal to the square A H, for the trapezium A B M L is similar and equivalent to the trapezium A I H O, and the triangle A B O is common. Again, the triangle A K E is half of the rectangle A E, and is equivalent to A I E or A B C, and K E is equal to A B; the angle K E N is equal to the angle B A C, and the angles E K N and A B O are right angles; therefore the triangle K E N is similar and equivalent to A B O, (Theor. 6. Cor. 3.) and the trapezium A L M B and the triangle K E N is equivalent to the square A I H B: because D G is parallel to C M, D N is parallel to C L, and N G is parallel to I M, the triangle D G N and C L M are similar; but they are also equivalent, for E N is equal to A O, which is equal to A L, and D N must be equal to C L: (Cor. 3. Theor. 6.) therefore the polygon C B N D and the triangles A K E and C L M are equivalent to the square C G, for the triangle A K E is equal to C F D, and G L M is equal to D N G, and consequently the polygon C B N D, and the trapezium A L M B, together with the triangles A K E, E K N, and C L M are equivalent to the square A C D E, or to the squares A I H B and C B G F together.

THEOREM XI.

If a right line A B *be divided into any two parts* A C, C B, *then will the square of the whole line* A B *be equal to the sum of the squares of the parts* A C *and* C B, *together with twice the rectangle of the parts.*

Upon A B construct the square A B D E, and make B G equal to A C, and draw C H parallel to B D, and G I parallel to E D: because A F is to the square A C, and F D is to the square of C B, and C G and I H are each equal to the rectangle of A C and C B:—the squares A F and F D together, with the rectangles I H and C G, make up the square A D.

THEOREM XII.

If a straight line A B *touches a circle, the straight line* C D *drawn from the centre to the point of contact shall be perpendicular to the line touching the circle.*

Through the centre C draw E F parallel to A B, and from E and F draw F A and F B perpendicular to E F or A B—because E F and A B are parallel, E A and F B are equal, (Def. 6.) and are the shortest lines which can possibly be drawn from the points E and F to A B: but A B touches the circumference only at the point D, therefore D is the nearest point of A B to the centre C; and C D, which is equal to the radius, must be also the shortest line which can be drawn from C to the line A B: consequently C D is equal to E A or F B, and is likewise a perpendicular to A B.

THEOREM XIII.

If a straight line E F *touches a circle, and from the point of contact* B *a straight line* B D *be drawn cutting the circle, the angle* D B F *made by this line with the line* E F, *shall be equal to the angle* B A D *in the alternate segment of the circle.*

Draw the diameter B A, and join D A; the angles B D A and A B F are right angles, therefore the angles B A D and A B D are together equal to a right angle: (Theor. 6.) but the angles A B D and D B F are also equal to a right angle, (Theor. 12.) consequently the angles B A D and A B D are equal to the angles D B F and A B D. Take the angle A B D from each, and B A D is equal to the angle D B F.—In like manner it may be demonstrated, that the angle C B F is equal to the angle B A C, and generally D B F is equal to the angles B A D or B G D, and C B F is equal to the angle B A C or B H C. (Theor. 5. Cor. 6.)

THEOREM XIV.

If two right lines A B *and* C D *intersect each other in a circle, the half of the sum of the intercepted arcs* A D *and* C B *measure the angle* C E A *or* B E D.

Draw D B; the angles A E D and D E B are equal to two right angles, and the angles D E B, E B D, and E D B are also equal to two right angles; (Theor. 6.) therefore by taking the angle D E B

from each, the angle A E D is equal to the angles E B D and E D B: but the angle E D B is measured by the half of the arc C B, and E B D by the half of the arc A D; (Theo. 5. Cor. 2.) consequently the angle A E D is measured by half the arc C B, together with half the arc A D.

PROPORTION.

EXPLANATION OF SIGNS.

\+ *Plus*, signifies the addition of the quantities between which it stands, as A+B, that is, B is to be added to A.

— *Minus*, signifies that the quantities before which it stands are to be subtracted, as A—B, that is, B is to be subtracted from A.

× *Into*, signifies that the quantities between which it stands are to be multiplied together, as A×B, is A multiplied by B, and is put in the form A B when each letter expresses a quantity.

÷ *Divided by*, signifies that the former is to be divided by the latter, as A÷B, that is, A is to be divided by B. Sometimes the division of quantities is also expressed by placing the divisor below the dividend, as $\frac{A}{B}$

= *Equal to*, signifies that the quantities between which it is placed are equal to each other, as A=B, that is, A is equal to B.

This sign, $\overline{\quad}$ called a *vinculum*, when placed over quantities, signifies that they are to be taken collectively, as $\overline{A+B+C}\times E$, that is, the sum of A, B, and C is to be multiplied by E.

The small *integers* ^{2}, ^{3}, ^{4}, &c. annexed to quantities, signify, that the quantity is to be multiplied that number of times by itself; as, $A^2=A\times A$, and is called the square of that quantity, or second power, $A^3=A\times A\times A$, and is called the cube, or third power, and $A^4=A\times A\times A\times A$, the biquadrate, or fourth power.

The signs $\sqrt[2]{}$, $\sqrt[3]{}$, $\sqrt[4]{}$, &c. placed before any quantity, as $\sqrt[2]{A}$, signify that the operation of extracting the root of that power is to be performed, as $\sqrt[2]{A^2}=A$, or $\sqrt[3]{A^3}=A$, &c.

DEFINITIONS.

1. If magnitudes or quantities be referred to one common standard of measure, the comparison of their respective values is called the *ratio* of these magnitudes. Thus if A is found to equal 5, and B=10, the ratio of A to B is as 5 is to 10.

2. If one quantity contain another any number of times, the first is called a *multiple* of the second. Thus B=10 is a multiple of A=5.

3. The first term of a ratio is called the *antecedent*, and the second the *consequent*; as in the ratio of A to B, A is the antecedent, and B the consequent. The ordinary expression for the ratio of two quantities, is made by setting the antecedent above the consequent, as $\frac{A}{B}$.

4. The multiple of a ratio is the product of each of the terms by the same quantity; as $\frac{A}{B}$ multiplied by C will stand $\frac{A\times C}{B\times C}$. The product of the antecedent becomes a new antecedent, and the product of the consequent a new consequent, having the same ratio to each other; as $\frac{A}{B}=\frac{2A}{2B}=\frac{3A}{3B}$ &c.

5. *Ratio of equality* is when the antecedent is equal to the consequent.

6. Four quantities are said to be *proportional*, which, when compared two and two, are found to have the same ratios; as in the quantities A, B, C, D; A is to B, as C to D, and A is to C, as B to D, or the ratio $\frac{A}{B}=\frac{C}{D}$ and $\frac{A}{C}=\frac{B}{D}$, and the proportion is thus expressed; A : B :: C : D, or A : C :: B : D. The first proportion is called *direct*, for the second term is consequent to the first, and the fourth a consequent to the third; and the second *alternate*, because the third term is consequent to the first and the fourth a consequent to the second.

THEOREM XV.

When four quantities a, b, c, d are proportional, the product of the extreme a and d is equal to the product of the means b and c.

If $a : b :: c : d$, then $\frac{a}{b}=\frac{c}{d}$ and $\frac{a}{b}=\frac{a\times d}{b\times d}$ also $\frac{c}{a}=\frac{c\times b}{d\times b}$ therefore $\frac{a\times d}{b\times d}=\frac{c\times b}{d\times b}$ (Def. 4.) this ratio put again in the first form, is $a\times d : b\times d :: c\times b : d\times b$. Hence since the consequents are equal, the antecedents are equal, and $a\times d=c\times b$.

Cor. 1.—If the first term be to the second as the second to the third, that is, as $a : b :: b : c$, the rectangle of the extreme is equal to the square of the mean, or $a\,c=b^2$.

Cor. 2.—If two rectangles are equal, their sides are reciprocally proportional; if the rectangle x is equal to the rectangle z, then $a\,b = c\,d$, and $a : c :: d : b$.

THEOREM XVI.

In any plain triangle A B C, *the adjoining sides* A B, A C, *are cut proportionally by a line* D E *drawn parallel to the other side* B C; *that is,* A D : D B :: A E : E C.

THROUGH A draw $b\,a$ parallel, and B b and C a perpendicular to B C. Bisect b A and A a in p and r, and draw $p\,q$ and $r\,s$ parallel to B b or C a, and join $d\,c$.—Because p A is equal to B q, and A r equal to s C, the triangle A $d\,p$ is similar and equal to the triangle B $d\,q$, and the triangle A $c\,r$ is similar and equal to the triangle C $c\,s$. (Theor. 6. Cor. 3.) Hence A d is equal to B d, A c is equal to c C, $p\,d$ is equal to $d\,q$, and $r\,c$ is equal to $c\,s$; consequently $p\,d$ is also equal to $r\,c$, and $d\,c$ is parallel to B C: but D E is parallel to $d\,c$; therefore in like manner it may be demonstrated, that like parts are cut off from A d and A c by the parallel D E, and A D : D B :: A E : E C.

Cor. 1.—Hence, when the sides A B, A C of a triangle are cut proportionally in D and E, the segments A D, A E, and D B, E C of those sides are proportional to the sides; and the line D E joining those sections, is parallel to the other side B C.

Cor. 2.—In equiangular triangles, as A D E, A B C, the sides adjacent to the equal angles are proportional, and also the sides opposite the equal angles are proportional.

THEOREM XVII.

In a right angled triangle, if a perpendicular A D *be drawn from the right angle* B A C *to the base* B C, *the triangles on each side of it are similar to the whole triangle, and to one another.*

Because the angles A D B and B A C are both right angles; the angle A B D is common to the triangles B A C and A B D, which therefore have the remaining angles B A D and B C A equal, (Theor. 6.) and the triangles B A C and A B D are equiangular.—In like manner it may be demonstrated—the triangle A C D is similar to the triangle B A C, and consequently the triangles A B D, A D C, and B A C are similar.

Cor.—Hence B D : A D :: A D : D C, or the perpendicular A D is a mean proportional to the segments upon the base B D and D C; and B D × D C = A D^2. (Theor. 15. Cor. 1.)

THEOREM XVIII.

If two chords A B *and* C D *intersect each other within a circle or without it, by being produced, the rectangle under the segments made by their intersection, and terminated by the circumference, are equal; that is,* B E × E A = D E × E C.

Join B C and D A; because the angle D E A and B E C are equal, (Theor. 2.) and the angle C B A and C D A are also equal, the triangle B E C is similar to the triangle D E A; (Theor. 5. Cor. 2.) therefore A E : C E :: D E : B E, or B E × A E = D E × C E. (Theor. 15.)

LOGARITHMS.

LOGARITHMS are a set of artificial numbers, arranged in tables, peculiarly adapted to facilitate the enumeration of natural numbers. Their properties are such, that the sum of the Logarithms corresponding to any two or more natural numbers, answers to the Logarithm of their product. The Logarithm of every number is expressed by one of the indices 0, 1, 2, 3, 4, 5, &c. with decimals annexed to each index · as the Log. of 1 is 0.0000000, of 10 is 1.0000000, of 100 is 2.0000000, of 1000 is 3.0000000, &c. Hence it appears, that the indices of the Logarithms form a series in Arithmetical progression, or have a common difference in each term, and the natural numbers answering to this series are in Geometrical progression, or every term is a certain multiple of the preceding one. In the following system of Logarithms, the numbers corresponding to every different integer of the indices are each a power of 10; for example, the Log. index 2 answers to $100=10^2$, and 3 answers to $1000=10^3$, &c. but if this series was changed by assuming the power of any other number corresponding to the same indices, another system of Log. would be formed, having the same peculiar properties as the above: for instance, if we had assumed the Geometrical series of numbers to be the powers of 8, then every different index of the Log. would correspond to a different power of 8; as the Log. index 1 would in this system answer to 8, and the

index 2 to $64=8^2$, and 3 to $512=8^3$, &c. so that as any number may be taken in place of 10, there may be an infinite number of different systems of Logarithms.

Since in the following system 0 is the Logarithm of 1, 1 of 10, 2 of 100, and 3 of 1000, the Logarithms of the numbers lying between 1 and 10 must be each greater than 0, and less than 1, therefore the Logarithm of these are expressed by decimals of 1; as.

No.	*Logarithms.*
2	0.3010300
3	0.4771213
4	0.6020600
5	0.6989700
6	0.7781513
7	0.8450980
8	0.9030900
9	0.9542425

The same may be shown of the Logarithms of numbers between 10 and 100, or between 100 and 1000; for those between 10 and 100 must be each greater than 1 and less than 2, and between 100 and 1000 each is greater than 2 and less than 3; as,

No.	*Logarithms.*	*No.*	*Logarithms.*
20	1.3010300	200	2.3010300
30	1.4771213	300	2.4771213
40	1.6020600	400	2.6020600
50	1.6989700	500	2.6989700
60	1.7781513	600	2.7781513
70	1.8450980	700	2.8450980
80	1.9030900	800	2.9030900
90	1.9542425	900	2.9542425

Note.—To those who wish to understand thoroughly the construction of Logarithms, I recommend for their perusal the Introduction to Hutton's Logarithmic Tables.

EXPLANATION OF TABLE I.

The first or left-hand column of the first page, which is marked N. contains the natural numbers from 1 to 100, and in the second column, marked L. are the corresponding Logarithms to each number.

The second, and all the remaining pages of this Table, are divided into eleven columns, the first of which is, as before, marked N. containing all the Numbers from 100 to 1000, and the remaining ten, marked 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, contain the corresponding Logarithms of all the numbers from 100 to 10,000.

The indices are not printed with any of the Logarithms, except those Numbers from 1 to 100, but the index of each must always be prefixed when used; it being understood, that the index 2 is to be put before the Tabular Logarithm for all numbers from 100 to 1000, and 3 from 1000 to 10,000, and so on, making the index always one less than the number of integer figures in the Number for which the Logarithm is taken. Also in the same columns the first figure after the index being the same for several lines, is not repeated except in column 0, when it changes, but which, likewise, must be always prefixed to the other figures—thus making every Logarithm of this Table to contain seven places besides the index.

There being a difference between the Logarithm of every different number, the parts of these differences are proportioned for every integer from 1 to 10: they are arranged under the Logarithms of each page, to which they respectively belong, and titled PROPORTIONAL PARTS. This Table has a column N. containing the Numbers, and another D. of the corresponding Differences; but the particular use of it will be better explained in the following examples.

Examples for finding the Logarithm of any given Number,

Ex. 1.—The Logarithms of all the Numbers of two places, that is, from 1 to 100, are found in the first page of the Table, in the column L.; thus if that of 16 is wanted, the Logarithm opposite that Number, viz. 1.2041200, is the one required.

Ex. 2.—To find the Logarithm of a number of three places, as 165. Find in the column N. the No. 165, and opposite, in column 0, is .2174839, to which prefix the index 2, and 2,2174839 is the Logarithm required.

Ex. 3.—To find the Logarithm of a number of four places, as 2768. Find the three first figures viz. 276, in column N. and in column 8 the name of the fourth figure is .4421661, to which prefix the index 3, and 3.4421661 is the Logarithm required.

Ex. 4.—To find the Log. of a number of five places, as 13458. Find, by Ex. 3, the Logarithm of the first four figures, viz. 1345—1287223, and subtract this Logarithm from the next highest, viz. .1290451, and the difference will be 3228; then take 8 tenths of this difference, or the tenth part of it multiplied by the fifth figure of the given Number, which add to the Logarithm found, as .1287223+2582=.1289805, to which prefix the index 4, and 4.1289805 is the Logarithm required. Instead of subtracting the whole Logarithms from each other for finding the difference, subtract only the last figure of the lower from the last of the higher, and the remainder is the fourth figure of the difference; and in the column N. of the Proportional Parts, find the first figures of the given number, or the nearest number to them, and in the same line in column D. is the three first figures, which prefixed to the fourth, gives the whole *Tabular Difference*: as for example, the subtracted last figures of the Logarithms of 1345 and 1346 is 8
and opposite 1347 in column D. of Proportional Parts is - 322
making - - - - - - 3228

the same as before. In the same manner the tabular difference of any two Logarithms may always readily be found by inspection.

Ex. 5.—To find the Logarithm of six, seven, and eight places.

First, for six figures, or 154396. By the last example find the Logarithm of the first five figures, viz. 15439, which is .1886192, then in the column N. of the Table of Proportional Parts, find the same number, or the next lowest, and in the same line take out the proportional part 169 in the column of the sixth figure. This added to the Logarithm found, as .1886102+169=5.1886261, is the Logarithm required.

Second, for seven figures, or 1543964. As above find the Logarithm for the first five figures, viz. 15439, and the Proportional Parts for 6 the sixth figure, and for 4 the seventh; of which add to the Logarithm found the whole for the sixth Proportional Part, and the tenth for the seventh: as,

15439	the Logarithm of is - -	1886191.6
Pro. Part for	6 - - - - -	169
Pro. Part for	4 - - - - -	11.2
1543964		6.1886372.

the Logarithm required.

Third, for eight figures, or 15439647. The same operation as before is to be performed for finding the Logarithm of seven figures, and add the hundredth of the Proportional Part for the eighth figure: as

15439	the Logarithm of is	.1886191.6
Pro. Part for	6 - - - - -	169
Do. for	4 - - - - -	11.2
Do. for	7 - - - - -	1.97
15439647		7.1886373.77

When it happens that the remainder or first figure cut off to the right

hand of the sum of the Logarithm after the Proportional Parts are added, is above 5, the last figure of the Logarithm is to be increased by unity; as in the last example, 7.1886378.7 will become 7.1886374.

The Logarithms of Decimal Numbers are found in the same manner as the preceding examples, by considering the Decimals as a whole number: but prefixing the value of the index according to the number of figures in the given integral only: as for example,

No. Decimals.		*Logarithms.*
1.5439647	——	0.1886374
15.439647	——	1.1886374
154.39647	——	2.1886374
1543.9647	——	3.1886374
15439.647	——	4.1886374
154396.47	——	5.1886374
1543964.7	——	6.1886374

In the case where the number for which the Logarithm is wanted is wholly a Decimal, the index of such becomes negative, and is marked by the sign *minus* over it. If there is no cypher between the first significant integer and the decimal point, the index is $\overline{1}$, and if one cypher it is $\overline{2}$, and if two cyphers it is $\overline{3}$, &c. it being a general rule to make the index for Decimals one more than the number of cyphers between the first significant figure and the decimal point; as,

Decimals.		*Logarithms.*
.15439647	——	$\overline{1}$.1886374
.015439647	——	$\overline{2}$.1886374
.0015439647	——	$\overline{3}$.1886374
.00015439647	——	$\overline{4}$.1886374
.000015439647	——	$\overline{5}$.1886374
0000015439647	——	$\overline{6}$.1886374

Ex. 6.—To find the Logarithm of a fraction, or a mixed number, as $\frac{50}{16}$. Reduce the vulgar fraction to a decimal, and find its Logarithm by the preceding examples for whole numbers; or subtract the Logarithm of the denominator from the Logarithm of the numerator, and the remainder will be the Logarithm wanted; as, $\frac{50}{16}$=3.125 Logarithm of is 0.4948500, which was required: or,

From the Logarithm of 50 is	——	1.698970
Subtract the Logar. of 16	——	1.204120
		0.494850

If a Vulgar Fraction be subjoined to a whole number, the whole expression may be reduced to a fraction, and the Logarithm of it thence found as above; as $17\frac{1}{5}=\frac{88}{5}$; or the fraction may be transformed into a decimal, and the Logarithm of the whole found as a decimal number; as $17\frac{1}{5}$=17.6: for,

From the Logarithm of 88	——	1.9444827
Subtract the Logar. of 5	——	0.6989700
		1.2455127

is the Logarithm of $17\frac{1}{5}$. Also, the Logarithm of 17.6=$17\frac{1}{5}$ is 1.2455127.

To find the Natural Number answering to any given Logarithm.

Find the first three figures of the given Logarithm after the index, or the three next lowest in column 0, and in the same line seek the given Logarithm, or the next lowest to it, and the number opposite in column N., are the three first figures of the number sought, and the number at the top of the column in which the given Logarithm, or its nearest is found, is the fourth.

If the index of the given Logarithm is 0, 1, or 2, the four figures

thus found, will be partly decimal, as the integral number must be marked off upon the left, according to the index: but if the index is 8, there will be a whole number. If the given Logarithm cannot be exactly found, subtract the next lowest from it, and divide the difference by the tenth parth of the Tabular Difference, (Ex. 4.) and the quotient will be the fifth figure of the number sought. Again, if there be a second remainder, seek the five figures already found, or the nearest number to them in column N. of Proportional Parts, and in the same line, seek also the number of the remainder, and the figure at the top of the column in which it is found is the sixth figure: but if this remainder is not exactly found, subtract the next Proportional Part from it, and this third remainder, by annexing a cypher, becomes a number, by which, in like manner, the seventh figure may be found; and if the fourth remainder cannot be exactly found in the same line, subtract the next lowest part from it, and by adding a cypher to the remainder, it becomes also a number, by which the eighth figure is found: as

Ex. 7.—To find the natural number answering to the Logarithm,

	7.1886374
Next lowest Logarithm of 1563	1883669
First Remainder	2715

which divided by the Tabular Difference, as 281.4)2715.0(9 the fifth figure of the number sought.

	25326
Second Remainder	182.4
6 the first Proportional Part	169
Third Remainder	134
4 the second Proportional Part	112
Fourth Remainder	220
7 the third Proportional Part	197

Hence the Natural Number found for the Logarithm 7.1886374

when put together is 15469647. The three last figures may be found otherwise thus; after finding the first five figures as above, find the Tabular difference answering to them, (Ex. 4.) and divide the second remainder by it.—Continue this division by adding cyphers, as far as the index of the Logarithm requires it to be made, and the quotient will be the figures sought: as

23)182. (647 the same as before.
1686
1340
1124
2160
1967

ARITHMETIC BY LOGARITHMS.

I. *MULTIPLICATION.*

ADD together the Logarithms of all the factors, and the sum is the Logarithm of their product.

If the indices are all positive, add to them the number carried from the decimal addition, and the sum is the index of the Logarithmic product: but if both positive and negative, the number carried from the decimal Addition is to be added to the positive indices, and the difference of the positive and negative to be put down for the index of the Logarithmic product with the sign of the greatest.

Ex. 8.—Multiply 26.04 by 6.325.

The Log. of 26.04 is	1.4156410
——— of 6.325 is	0.8010605
Product 164.703 -	2.2167015

Ex. 9.—Multiply 2.603, 673.4 and .2638 together.

2.603 Log. of is -	0.4154742
673.4 ——— -	2.8282731
.02638 ——— -	$\overline{2}$.4212748
Product 46.24045 -	1.6650221

Ex. 10.—Multiply 2.36, 2.48, .062 and .002 together.

2.36 Log. of is -	0.3729120
2.48 ——— -	0.3944517
.062 ——— -	$\overline{2}$.7923917
.002 ——— -	$\overline{3}$.3010300
Product .00072574	$\overline{4}$.8607854

Ex. 11.—Mult. 32684 by 67324.

32684 Log. of is -	4.5143352
67324 ——— -	4.8281699
Product 2200417610	9.3425051

It is necessary to observe, that the number answering to any given Logarithm found by Table I. is true to eight or nine figures, but cannot be depended on farther; for, by the last example, the true product is 2200417616, so that in this case it is true to the ninth figure. Tables which would give correct answers to more figures, would require the Logarithmic decimal extended to a greater number of places, and hence would be much more voluminous than this

II. *DIVISION.*

From the Logarithmic decimal of the dividend subtract the Logarithmic decimal of the divisor—the remainder will be the Logarithmic decimal of the quotient.

But change the sign of the index of the divisor from plus to minus, or from minus to plus, and the sum of the indices of the same sign, or the difference when of different signs with the sign of the greater, is to be set down for the index of the Logarithm of the quotient.

Also, what one is carried from the Logarithmic decimal, add it to the index of the divisor when that index is plus, but subtract it when minus, and the index thus found is to be changed as before.

Ex. 12.—Divide 1363 by 1268.

Divid. 1363	Log. of is	3.1344959
Divis. 1268	———	3.1031196
1.0749	- -	0.0313763

Ex. 13.—Divide 3462 by 6924.

Divid. 3462	Log. of is	3.5396271
Divis. 6924	———	3.8403571
.5	- -	$\overline{1}$.6992700

Here the Logarithm of the divisor is positive, and 1 carried from the decimal being added to its index, and then changed to the negative, the difference between the indices becomes $\overline{1}$.

Ex. 14.—Divide 28.23 by 344.67.

Divid. 28.23	Log. of is	1.4507109
Divis. 344.67	———	2.5374035
.08179044	-	$\overline{2}$.9133074

Ex. 15.—Divide .6823 by 234.6

Divid. .6823	Log. of is	$\overline{1}$.8339754
Divis. 234.6	———	2.4703280
.002915	- -	$\overline{3}$.4636474

In this last example, by changing the positive index of the divisor to negative, it becomes of the same sign of the dividend, or both negative, and the sum is $\overline{3}$.

III. *PROPORTION.*

Add the Logarithms of the second and third terms together, and from the sum subtract the Logarithm of the first by the rules of the preceding examples—the remainder will be the Log. of the 4th term.

Ex. 16.—Find a fourth proportional to 36.32, 3.648, and 423.6.

As 36.32 Log. of	1.5601458
is to 3.648 -	0.5620548
so is 423.6 -	2.6269560
	3.1890108
to the 4th term 42547—	1.6288650

Ex. 17.—Find a fourth proportional to 2.46, 0.23, and 1.35.

As 2.46 Log. of	0.3909351
is to 0.23 -	$\overline{1}$.3617270
so is 1.35 -	0.1303338
	$\overline{1}$.4920608
4th term 1.2621	$\overline{1}$.1011257

Instead of subtracting the Logarithm of the first term, the arithmetical complement of it may be added; and the sum, after subtracting 10 from the index, is the Logarithm of the fourth term. The *Arithmetical Complement* of any Logarithm is found by subtracting it from 10.—Hence the 16th and 17th examples will stand.

As 36.32 Arith Comp.	8.4398542	As 2.46 Arith. Comp.	10.6090649
is to 3.648 Log. of is	0.5620548	is to 0.92 Log. of is	$\bar{1}$.9637270
so is 423.6 ———	2.6269560	so is 1.25 ———	0.1303338
to the 4th term 42.547	1.6288650	4th term - -	0.1011257

The easiest method of finding the arithmetical complement is to begin at the left hand, and subtract each figure from 9, except the last significant figure upon the right, which must be subtracted from 10; but when the index is negative, add it to 9, and subtract the rest as before.

IV. *INVOLUTION.*

Multiply the Logarithm of the given number by the index of the power to be raised, and the product is the Logarithm of the power required.

If the index of the Logarithm of the given number happens to be negative, the product will be also negative; but the number carried from the Logarithm decimal to the index is positive, so that the difference will be the index of the Logarithm product, and to be set down with the sign of the greater.

Ex. 18.—Find the square or 2d power of 26.23.

26.23 Log. of is -	1.4187983
index of the power -	2
power 688.0129 -	2.8375966

Ex. 19.—To find the cube or 3d power of .123.

.123 Log. of is -	$\bar{1}$.0899051
index of the power -	3
power .001860867	$\bar{3}$.2697153

Ex. 20.—Find the biquadrate, or 4th power of .26.

.26 Log. of is -	$\bar{1}$.4149733
index of the power -	4
power .00456976 -	$\bar{3}$.6598932

Ex. 21.—Find the 16th power of .0012.

.0012 Log. of is -	$\bar{3}$.0791812
index of the power -	16
	4750872
	0791812
product of the Log. Dec.	1.2668992
product of the index	48.
.000000000000000000000000000000000000000000000000	47.2668992

0184884 the power required.

In the 20th example, after multiplying the Logarithmic Decimal by 4, there is one to carry to the index; but the index being here negative, the difference of the number carried, which is positive, and the product of Logarithmic index, and index of the power, is set down for the Logarithmic index of the power sought, with the sign minus, which is the sign of the greater.

In such cases as example 21st, in which it is necessary to put down the whole operation of multiplication, the figures of this product which are cut off to the right hand are the same number of Decimals as in the Logarithm, and the figure or figures upon the left of the point, are those which are to be carried to the index, and managed as shown above.

EVOLUTION.

Divide the Logarithm of the power by the index of the root, and the quotient will be the Logarithm required.

Ex. 22.—Find the square root of 688.0129.

688.0129 Log. of is 2.8375966
which divided by 2, the index of the power, the quotient is - 1.4187983
and the number corresponding to this is 26.23, the root required.

Ex. 23.—Find the cube root of .001860867.

.001860867 Log. of is 3.2697153
which divided by 3, the index of the power, the quotient is - $\bar{1}$.0899051
and the number answering is .123, the root sought.

When the index happens to be negative, and the index of the Logarithm of the given number cannot be exactly divided by it, add one or more to the Logarithmic index, by which an exact quotient may be formed, and carry the same number as ten to the decimal places when dividing the rest, and the quotient is the Logarithm of the root: as,

Ex. 24.—Find the 16th root of .00000000000000000000000000000000000000000000184884,

the Log. of which is $\overline{47}$.2668992
to the index of which add 1, and divide by 16, the quotient is $\bar{3}$.0791812
the number answering to this is .0012, which is the root required.

In this example, by adding 1 to the index, the first figure of the quotient is $\bar{3}$, and carrying the 1 as 10 to the decimal the other part of the quotient becomes .0791812 as above.

TABLE I.

CONTAINING THE

LOGARITHMS OF NUMBERS,

From 1 to 10,000.

Num.	Log.	Num.	Log.	Num.	Log.
1	0 0000000	34	1.5314789	67	1.8260748
2	0.3010300	35	1 5440680	68	1.8325089
3	0.4771213	36	1.5563025	69	1 8388491
4	0 6020600	37	1 5682017	70	1.8450980
5	0.6989700	38	1 5797836	71	1 8512583
6	0.7781513	39	1 5910646	72	1 8573325
7	0.8450980	40	1.6020600	73	1 8633229
8	0 9030900	41	1 6127839	74	1 8692317
9	0.9542425	42	1 6232493	75	1.8750618
10	1 0000000	43	1 6334685	76	1 8808136
11	1 0413927	44	1 6434527	77	1 8864907
12	1.0791812	45	1 6532125	78	1 8920946
13	1.1139434	46	1.6627578	79	1 8976271
14	1 1461280	47	1.6720979	80	1 9030900
15	1.1760913	48	1.6812412	81	1 9084850
16	1.2041200	49	1 6901961	82	1 9138139
17	1.2304489	50	1 6989700	83	1 9190781
18	1 2552725	51	1 7075702	84	1 9242793
19	1 2787536	52	1 7160033	85	1 9294189
20	1.3010300	53	1 7242759	86	1 9344985
21	1 3222193	54	1 7323938	87	1 9395193
22	1 3424227	55	1 7403627	88	1 9444827
23	1 3617278	56	1 7481880	89	1 9493900
24	1 3802112	57	1 7558749	90	1 9542425
25	1 3979400	58	1 7634280	91	1 9590414
26	1.4149733	59	1 7708520	92	1 9637878
27	1 4313638	60	1.7781513	93	1 9684829
28	1.4471580	61	1.7853298	94	1 9731279
29	1 4623980	62	1 7923917	95	1 9777236
30	1 4771213	63	1 7993405	96	1 98227.2
31	1.4913617	64	1 8061800	97	1.9867717
32	1.5051500	65	1.8129134	98	1 9912261
33	1.5185139	66	1 8195439	99	1.9956352

N.	0	1	2	3	4	5	6	7	8	9
100	0000000	004341	008677	013009	017337	021661	025980	030295	034605	038912
101	043214	047512	051805	056094	060380	064660	068937	073210	077478	081743
102	086002	090257	094509	098756	103000	107239	111474	115704	119931	124154
103	128372	132587	136797	141003	145205	149403	153596	157788	161974	166156
104	170333	174507	178677	182843	187005	191163	195317	199467	203613	207755
105	211893	216027	220157	224284	228406	232525	236639	240750	244857	248960
106	253059	257154	261245	265333	269416	273496	277572	281644	285713	289777
107	293838	297895	301948	305997	310043	314085	318123	322157	326186	330214
108	334238	338257	342273	346285	350293	354297	358298	362295	366289	370279
109	374265	378248	382228	386202	390173	394141	398104	402066	406023	409977
110	413927	417873	421816	425755	429691	433623	437551	441476	445398	449315
111	453230	457141	461048	464952	468852	472 49	476642	480532	484418	488301
112	492180	496056	499929	503798	507663	511525	515384	519239	523091	526939
113	530784	534626	538464	542299	546131	549959	553783	557605	561423	565237
114	569049	572856	576661	580462	584260	588055	591846	595634	599419	603200
115	606978	610753	614525	618293	622058	625820	629578	633334	637086	640834
116	644580	648322	652061	655797	659530	663259	666986	670709	674428	678145
117	681859	685569	689276	692980	696681	700379	704073	707765	711453	715138

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
10000	434	43	87	130	174	217	260	304	347	391
10020	433	43	87	130	173	217	260	303	346	390
10040	432	43	86	130	173	216	259	302	346	389
10060	431	43	86	129	172	216	259	302	345	388
10080	430	43	86	129	172	215	258	301	344	387
10110	429	43	86	129	172	215	257	300	343	386
10130	428	43	86	128	171	214	257	300	342	385
10160	427	43	85	128	171	214	256	299	342	384
10180	4	43	85	128	170	213	256	298	341	383
10200	425	43	85	128	170	213	255	298	340	383
10230	424	42	85	127	170	212	254	297	339	382
10250	423	42	85	127	169	21	254	296	3 8	381
10280	422	42	84	127	169	211	253	295	338	380
10300	421	42	84	126	168	211	253	295	337	379
11330	420	42	84	126	168	210	252	294	336	378
10350	419	42	84	126	168	210	251	293	335	377
10370	418	42	84	125	167	209	251	293	334	376
10400	417	42	83	125	167	209	250	292	334	375
10430	416	42	83	125	166	208	250	291	333	374
10450	415	12	83	125	166	208	249	291	332	374
10480	414	41	83	124	166	207	248	290	331	373
10500	413	41	83	124	165	207	248	289	330	372
10530	412	41	82	124	165	206	247	288	330	371
10560	411	41	82	123	164	206	247	288	329	370
10580	410	41	82	123	164	205	246	287	328	369
10610	409	41	82	123	164	205	245	286	327	368
10630	408	41	82	122	163	204	245	286	326	367
10660	407	41	81	122	163	204	244	285	326	366
10680	406	41	81	122	162	203	244	284	325	365
10710	405	41	81	122	162	203	243	284	324	365
10730	404	40	81	121	162	202	242	283	323	364
10760	403	40	81	121	161	202	242	282	322	363
10790	402	40	80	121	161	201	241	281	322	362

N.	D	1	2	3	4	5	6	7	8	9
10820	401	40	80	120	160	201	241	281	321	361
10850	400	40	80	120	160	200	240	280	320	360
10880	399	40	80	120	160	200	239	279	319	359
10900	398	40	80	119	159	199	239	279	318	358
10930	397	40	79	119	159	199	238	278	318	357
10950	396	40	79	119	158	198	238	277	317	356
10980	395	40	79	119	158	198	237	277	316	356
11010	394	39	79	118	158	197	236	276	315	355
11040	393	39	79	118	157	197	236	275	314	354
11070	392	39	78	118	157	196	235	274	314	353
11090	391	39	78	117	156	196	235	274	313	352
11120	390	39	78	11	156	195	234	273	312	351
11150	389	39	78	117	156	195	233	272	311	350
11180	388	39	78	116	155	194	233	272	310	349
11210	387	39	77	116	155	194	232	271	310	348
11240	386	39	77	116	154	193	232	270	309	347
11270	385	39	77	116	154	193	231	270	308	347
11300	384	38	77	115	154	192	230	269	307	346
11330	383	38	77	115	153	192	230	268	306	345
11360	382	38	76	115	153	191	229	267	306	344
11390	381	38	76	114	152	191	229	267	305	343
11420	380	38	76	114	152	190	228	266	304	342
11450	379	38	76	114	152	190	227	265	303	341
11480	378	38	76	113	151	189	227	265	302	340
11510	377	38	75	113	151	189	226	264	302	339
11540	376	38	75	113	150	188	226	263	301	338
11580	375	38	75	113	150	188	225	263	300	338
11610	374	37	75	112	15	187	224	262	299	337
11640	373	37	75	112	149	187	224	261	298	336
11670	372	37	74	112	149	186	223	260	298	335
11700	371	37	74	111	148	186	223	260	297	334
11730	370	37	74	111	148	185	222	259	296	333
11760	369	37	74	111	148	185	221	258	295	332

N.	0	1	2	3	4	5	6	7	8	9
118	0718820	722499	726145	729877	733517	737184	740847	744507	748164	751819
119	755470	759118	762762	766404	770043	773679	777312	780942	784568	788192
120	791812	795430	799044	802656	806265	809870	813473	817073	820669	824263
121	827854	831441	835026	838608	842187	845763	849336	852906	856473	860037
122	863598	867157	870713	874265	877814	881361	884905	888446	891984	895519
123	899051	902581	906107	909631	913152	916670	920185	923697	927206	930713
124	934217	937718	941216	944711	948204	951694	955180	958665	962146	965624
125	969100	972573	976043	979511	982975	986437	989896	993353	996806	000257
126	003705	007151	010593	014034	017471	020905	024337	027766	031193	034616
127	038037	041456	044871	048284	051694	055102	058507	061909	065309	068705
128	072100	075491	078880	082267	085650	089031	092410	095785	099159	102529
129	105897	109262	112625	115985	119343	122698	126050	129400	132747	136092
130	139434	142773	146110	149444	152776	156105	159432	162756	166077	169396
131	172713	176027	179338	182647	185954	189258	192559	195858	199154	202448
132	205739	209028	212315	215598	218880	222159	225435	228709	231981	235250
133	238516	241781	245042	248301	251558	254813	258065	261314	264561	267806
134	271048	274288	277525	280760	283993	287223	290451	293676	296899	300119
135	303338	306553	309767	312978	316187	319393	322597	325798	328998	332195
136	335389	338581	341771	344959	348144	351327	354507	357685	360861	364034
137	367206	370375	373541	376705	379867	383027	386184	389339	392492	395643
138	398791	401937	405080	408222	411361	414498	417632	420765	423895	427022

PROPORTIONAL PARTS.

N.	D.	1	2	3	4	5	6	7	8	9
11790	368	37	74	110	147	184	221	258	294	331
11820	367	37	73	110	147	184	220	257	294	330
11850	366	37	73	110	146	183	220	256	293	329
11880	365	37	73	110	146	183	219	256	292	329
11920	364	36	73	109	146	182	218	255	291	328
11950	363	36	73	109	145	182	218	254	290	327
11980	362	36	72	109	145	181	217	253	290	326
12020	361	36	72	108	144	181	217	253	289	325
12050	360	36	72	108	144	180	216	252	288	324
12080	359	36	72	108	144	180	215	251	287	323
12120	358	36	72	107	143	179	215	251	286	322
12160	357	36	71	107	143	179	214	250	286	321
12190	356	36	71	107	142	178	214	249	285	320
12230	355	36	71	107	142	178	213	249	284	320
12260	354	35	71	106	142	177	212	248	283	319
12280	353	35	71	106	141	177	212	247	282	318
12320	352	35	70	106	141	176	211	246	282	317
12360	351	35	70	105	140	176	211	246	281	316
12400	350	35	70	105	140	175	210	245	280	315
12430	349	35	70	105	140	175	209	244	279	314
12470	348	35	70	104	139	174	209	244	278	313
12500	347	35	69	104	139	174	208	243	278	312
12540	346	35	69	104	138	173	208	242	277	311
12580	345	35	69	104	138	173	207	242	276	311
12620	344	34	69	103	138	172	206	241	275	310
12650	343	34	69	103	137	172	206	240	274	309
12690	342	34	68	103	137	171	205	239	274	308
12720	341	34	68	102	136	171	205	239	273	307

N.	D.	1	2	3	4	5	6	7	8	9
12750	340	34	68	102	136	170	204	238	272	306
12790	339	34	68	102	136	170	203	237	271	305
12830	338	34	68	101	135	169	203	237	270	304
12870	337	34	67	101	135	169	202	236	270	303
12910	336	34	67	101	134	168	202	235	269	302
12950	335	34	67	101	134	168	201	235	268	302
12980	334	33	67	100	134	167	200	234	267	301
13020	333	33	67	100	133	167	200	233	266	300
13060	332	33	66	100	133	166	199	232	266	299
13100	331	33	66	99	132	166	199	232	265	298
13140	330	33	66	99	132	165	198	231	264	297
13180	329	33	66	99	132	165	197	230	263	296
13220	328	33	66	98	131	164	197	230	262	295
13260	327	33	65	98	131	164	196	229	262	294
13300	326	33	65	98	130	163	196	228	261	293
13340	325	33	65	98	130	163	195	228	260	293
13380	324	32	65	97	130	162	194	227	259	292
13420	323	32	65	97	129	162	194	226	258	291
13470	322	32	64	97	129	161	193	225	258	290
13510	321	32	64	96	128	161	193	225	257	289
13550	320	32	64	96	128	160	192	224	256	288
13590	319	32	64	96	128	160	191	223	255	287
13630	318	32	64	95	127	159	191	223	254	286
13670	317	32	63	95	127	159	190	222	254	285
13710	316	32	63	95	126	158	190	221	253	284
13760	315	32	63	95	126	158	189	221	252	284
13810	314	31	63	94	126	157	188	220	251	283
13860	313	31	63	94	125	157	188	219	250	282

N.	0	1	2	3	4	5	6	7	8	9
139	1430148	433271	436392	439511	442628	[illegible]	448864	451964	455073	458177
140	461280	464381	467480	470577	473671	[illegible]	[illegible]	482941	486027	489110
141	492191	495270	498347	501422	504494	[illegible]	[illegible]	513699	516763	519824
142	[illegible]	[illegible]	528941	528996	532049	535100	[illegible]	[illegible]	544240	547282	550322
143	553360	556396	559430	562462	565492	[illegible]	[illegible]	574568	577589	580608
144	[illegible]	[illegible]	590640	593653	596663	599672	[illegible]	604685	607686	610684
145	613680	616674	619666	622656	625644	[illegible]	[illegible]	634596	637575	640553
146	643529	646502	649474	652443	655411	[illegible]	[illegible]	664901	667861	670818
147	673173	[illegible]	679078	682027	684975	[illegible]	[illegible]	693805	696744	699682
148	702618	705551	[illegible]	711412	714339	[illegible]	[illegible]	723110	726029	728947
149	731863	734776	737689	740598	743506	[illegible]	[illegible]	752218	755118	758016
150	760913	763807	766699	769590	772478	[illegible]	[illegible]	781133	784013	786892
151	789769	792645	795518	798389	801259	[illegible]	[illegible]	809856	812718	815578
152	818436	821292	824147	826999	829850	[illegible]	[illegible]	838390	841234	844075
153	846914	849752	852588	855422	858254	[illegible]	[illegible]	866739	869563	872386
154	875207	878026	880844	883659	886473	[illegible]	[illegible]	894903	897710	900514
155	903317	906118	908917	911715	914510	[illegible]	920096	922886	925675	928461
156	931246	934029	936810	939590	942367	[illegible]	[illegible]	950690	953461	956229
157	958997	961762	964525	967287	970047	[illegible]	[illegible]	978317	981070	983821
158	986571	989319	992065	994809	997552	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
159	013971	016702	019431	022158	024883	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
160	041200	043913	046625	049335	052044	[illegible]	[illegible]	060159	062860	065560
161	068259	070955	073650	076344	079035	[illegible]	[illegible]	087100	089785	092468
162	095150	097830	100509	103185	105860	[illegible]	[illegible]	113876	116544	119211
163	121876	124540	127202	129862	132521	[illegible]	[illegible]	140487	143139	145790
164	148438	151086	153732	156376	159018	[illegible]	[illegible]	166936	169572	172207

PROPORTIONAL PARTS.

N.	D.	1	2	3	4	5	6	7	8	9
13900	312	31	62	94	125	156	187	218	250	281
13950	311	31	62	93	124	156	187	218	249	280
13990	310	31	62	93	124	155	186	217	248	279
14030	309	31	62	93	124	155	185	216	247	278
14080	308	31	62	92	123	154	185	216	246	277
14130	307	31	61	92	123	154	184	215	246	276
14170	306	31	61	92	122	153	184	214	245	275
14220	305	31	61	92	122	153	183	214	244	275
14270	304	30	61	91	122	152	182	213	243	274
14310	303	30	61	91	121	152	182	212	242	273
14360	302	30	60	91	121	151	181	211	242	272
14410	301	30	60	90	120	151	181	211	241	271
14450	300	30	60	90	120	150	180	210	240	270
14500	299	30	60	90	120	150	179	209	239	269
14550	98	30	60	89	119	149	179	209	238	268
14600	297	30	59	89	119	149	178	208	238	267
14650	296	30	59	89	118	148	178	207	237	266
14700	295	30	59	89	118	148	177	207	236	266
14750	294	29	59	88	118	147	176	206	236	265
14800	293	29	59	88	117	147	176	205	234	264
14850	292	29	58	88	117	146	175	204	234	263
14900	291	29	58	87	116	146	175	204	233	262
14950	290	29	58	87	116	145	174	203	232	261
15000	289	29	58	87	116	145	173	202	231	260
15050	288	29	58	86	115	144	173	202	230	259

N.	D.	1	2	3	4	5	6	7	8	9
15100	287	29	57	86	115	144	172	201	230	258
15150	286	29	57	86	114	143	172	200	229	257
15210	285	29	57	86	114	143	171	200	228	257
15270	284	28	57	85	114	142	170	199	227	256
15320	283	28	57	85	113	142	170	198	226	255
15380	282	28	56	85	113	141	169	197	226	254
15420	281	28	56	84	112	141	169	197	225	253
15470	280	28	56	84	112	140	168	196	224	252
15540	279	28	56	84	112	140	167	195	223	251
15600	278	28	56	83	111	139	167	195	222	250
15650	277	28	55	83	111	139	166	194	222	249
15700	276	28	55	83	110	138	166	193	221	248
15750	275	28	55	83	110	138	165	193	220	248
15810	274	27	55	82	110	137	164	192	219	247
15870	273	27	55	82	109	137	164	191	218	246
15920	272	27	54	82	109	136	163	190	218	245
15990	271	27	54	81	108	136	163	190	217	244
16060	270	27	54	81	108	135	162	189	216	243
16110	269	27	54	81	108	135	161	188	215	242
16180	268	27	54	80	107	134	161	188	214	241
16240	267	27	53	80	107	134	160	187	214	240
16290	266	27	53	80	106	133	160	186	213	239
16350	265	27	53	80	106	133	159	186	212	239
16400	264	26	53	79	106	132	158	185	211	238

N.	0	1	2	3	4	5	6	7	8	9
166	176630	177471	180100	[illegible]	185358	187980	190603	193225	195845	198466
168	201081	203696	206310	[illegible]	211533	214142	216750	219356	221960	224563
167	227165	229764	232363	[illegible]	237555	240148	242740	245331	247920	250507
168	253093	255677	258260	[illegible]	263421	266999	268576	271151	273724	276296
169	278867	281436	284004	[illegible]	289134	291697	294256	296815	299377	301934
170	304489	307043	309596	[illegible]	314696	317246	319790	322335	324879	327421
171	329961	332500	335039	[illegible]	340106	342641	345173	347703	350232	352759
172	355284	357809	360332	[illegible]	365373	367891	370408	372923	375437	377950
173	380461	382971	385480	[illegible]	390491	392995	395497	397998	400498	402996
174	405493	407988	410483	[illegible]	415465	417954	420442	422929	425414	427898
175	430380	432861	435341	[illegible]	440296	442771	445245	447718	450189	452658
176	455127	457594	460059	[illegible]	464986	467447	469907	472365	474823	477278
177	479733	482186	484637	[illegible]	489536	491984	494430	496874	499318	501759
178	504200	506639	509077	[illegible]	513949	516382	518815	521246	523675	526103
179	528530	530956	533380	[illegible]	538224	540645	543063	545481	547897	550312
180	552725	555137	557548	[illegible]	562367	564772	567177	569582	571984	574386
181	576786	579185	581582	[illegible]	586373	588766	591158	593549	595939	598327
182	600714	603099	605484	[illegible]	610248	612629	615008	617385	619762	622137
183	624511	626883	629255	[illegible]	633993	636361	638727	641092	643455	645817
184	648178	650536	652896	[illegible]	657609	659964	662317	664669	667020	669369
185	671717	674064	676410	[illegible]	681097	683439	685780	688119	690457	692794
186	695129	697464	699797	[illegible]	704459	706788	709116	711443	713769	716093
187	718416	720738	723058	[illegible]	727696	730013	732326	734643	736956	739268
188	741578	743886	746196	[illegible]	750809	753114	755417	757719	760020	762320
189	764618	766915	769211	[illegible]	773800	776092	778383	780672	782962	785250
190	787536	789821	792105	[illegible]	796669	798950	801229	803507	805784	808059
191	810334	812607	814879	[illegible]	819419	821688	823955	826221	828486	830750
192	833012	835274	837534	[illegible]	842051	844307	846563	848817	851070	853322
193	855573	857823	860071	[illegible]	864565	866810	869054	871296	873538	875778
194	878017	880255	882492	[illegible]	886963	889196	891428	893660	895890	898118
195	900346	902573	904798	[illegible]	909246	911468	913689	915908	918127	920344

PROPORTIONAL PARTS.

N.	D.	1	2	3	4	5	6	7	8	9
16490	263	26	53	79	105	132	158	184	210	237
16560	262	26	52	79	105	131	157	183	210	236
16610	261	26	52	78	104	131	157	183	209	235
16680	260	26	52	78	104	130	156	182	208	234
16720	259	26	52	78	104	130	155	181	207	233
16790	258	26	52	77	103	129	155	181	206	232
16860	257	26	51	77	103	129	154	180	206	231
16920	256	26	51	77	102	128	154	179	205	230
16990	255	26	51	77	102	128	153	179	204	230
17060	254	25	51	76	102	127	152	178	203	229
17120	253	25	51	76	101	127	152	177	202	228
17200	252	25	50	76	101	126	151	176	202	227
17270	251	25	50	75	100	126	151	176	201	226
17340	250	25	50	75	100	125	150	175	200	225
17400	249	25	50	75	100	125	149	174	199	224
17480	248	25	50	74	99	124	149	174	198	223
17540	247	25	49	74	99	124	148	173	198	222
17610	246	25	49	74	98	123	148	172	197	221
17680	245	25	49	74	98	123	147	172	196	221
17760	244	24	49	73	98	122	146	171	195	220
17830	243	24	49	73	97	122	146	170	194	219

N.	D.	1	2	3	4	5	6	7	8	9
17910	242	24	48	73	97	121	145	169	194	218
17980	241	24	48	72	96	121	145	169	193	217
18050	240	24	48	72	96	120	144	168	192	216
18120	239	24	48	72	96	120	143	167	191	215
18200	238	24	48	71	95	119	143	167	190	214
18270	237	24	47	71	95	119	142	166	190	213
18350	236	24	47	71	94	118	142	165	189	212
18430	235	24	47	71	94	118	141	165	188	212
18510	234	23	47	70	94	117	140	164	187	211
18600	233	23	47	70	93	117	140	163	186	210
18670	232	23	46	70	93	116	139	162	186	209
18750	231	23	46	69	92	116	139	162	185	208
18830	230	23	46	69	92	115	138	161	184	207
18910	229	23	46	69	92	115	137	160	183	206
18990	228	23	46	68	91	114	137	160	182	20
19080	227	23	45	68	91	114	136	159	182	204
19170	226	23	45	68	90	113	136	158	181	20
19250	225	23	45	68	90	113	135	158	180	20
19340	224	22	45	67	90	112	134	157	179	20
19430	223	22	45	67	89	112	134	156	178	201
19510	222	22	44	67	89	111	133	155	178	200

N.	0	1	2	3	4	5	6	7	8	9
196	2922561	924776	926990	929203	931415	[illegible]	[illegible]835	938044	940251	[illegible]
197	944662	946866	949069	951271	953471	[illegible]	[illegible]869	960067	962263	[illegible]
198	966652	968845	971037	973227	975417	[illegible]	[illegible]792	981979	984164	986348
199	988531	990713	992893	995073	997252	[illegible]				
	3						[illegible]605	003781	005955	008128
200	010300	012471	014641	016809	018977	021144	[illegible]309	025474	027637	029799
201	031961	034121	036280	038438	040595	042751	[illegible]905	047059	049212	051363
202	053514	055663	057812	059959	062105	064250	[illegible]394	068537	070680	072820
203	074960	077099	079237	081374	083509	085644	[illegible]778	089910	092042	094172
204	096302	098430	100557	102684	104809	106933	[illegible]056	111178	113300	115420
205	117539	119657	121774	123889	126004	128118	[illegible]231	132343	134454	136563
206	138672	140780	142887	144992	147097	149201	[illegible]303	153405	155505	157605
207	159703	161801	163898	165993	168088	170181	[illegible]273	174365	176455	178545
208	180633	182721	184807	186893	188977	191061	[illegible]143	195224	197305	199384
209	201463	203540	205617	207692	209767	211840	[illegible]913	215984	218055	220124
210	222193	224261	226327	228393	230457	232521	234584	236645	238706	240766
211	242825	244882	246939	248995	251050	253104	255157	257209	259260	261310
212	263359	265407	267454	269500	271545	273589	275633	277675	279716	281757
213	283796	285834	287872	289909	291944	293980	296012	298045	300077	302108
214	304138	306167	308195	310222	312248	314273	316297	318320	320343	322364
215	324385	326404	328423	330440	332457	334473	336488	338501	340514	342526
216	344538	346548	348557	350565	352573	354579	356585	358589	360593	362596
217	364597	366598	368598	370597	372595	374593	376589	378584	380579	382572
218	384565	386557	388547	390537	392526	394514	396502	398488	400473	402458
219	404441	406424	408405	410386	412366	414345	416323	418301	420277	422252
220	424227	426200	428173	430145	432116	434086	436055	438023	439991	441957
221	443923	445887	447851	449814	451776	453737	455698	457657	459615	461573
222	463530	465486	467441	469395	471348	473300	475252	477202	479152	481101
223	483049	484996	486942	488887	490832	492775	494718	496660	498601	500541
224	502480	504419	506356	508293	510229	512163	514098	516031	517963	519895
225	521825	523755	525684	527612	529539	531465	533391	535316	537239	539162
226	541084	543006	544926	546846	548764	550682	552599	554515	556431	558345
227	560259	562171	564083	565994	567905	569814	571723	573630	575537	577443
228	579348	581253	583156	585059	586961	588862	590762	592662	594560	596458
229	598355	600251	602146	604041	605934	607827	609719	611610	613500	615390
230	617278	619166	621053	622939	624825	626709	628593	630476	632358	634239

PROPORTIONAL PARTS

N.	D.	1	2	3	4	5	6	7	8	9
19600	221	22	44	66	88	111	133	155	177	199
19690	220	22	44	66	88	110	132	154	176	198
19780	219	22	44	66	88	110	131	153	175	197
19860	218	22	44	65	87	109	131	153	174	196
19960	217	22	43	65	87	109	131	152	174	196
20050	216	22	43	65	86	108	130	151	173	194
20150	215	22	43	65	86	108	129	151	172	194
20250	214	22	43	64	86	107	128	150	171	193
20340	213	21	42	64	85	107	128	149	170	192
20440	212	21	42	64	85	106	127	148	170	191
20530	211	21	42	63	84	106	127	148	169	190
20620	210	21	42	63	84	105	126	147	168	189
20740	209	21	42	63	84	105	125	146	167	188
20840	208	21	41	62	83	104	125	146	166	187
20960	207	21	41	62	83	104	124	145	166	186
21050	206	21	41	62	82	103	124	144	165	185
21110	205	21	41	62	82	103	123	144	164	185

N.	D.	1	2	3	4	5	6	7	8	9
21240	201	20	41	61	82	102	122	143	163	183
21340	203	20	41	61	81	102	122	142	162	183
21440	202	20	40	61	81	101	121	141	162	182
21530	201	20	40	60	80	101	121	141	161	181
21650	200	20	40	60	80	100	120	140	160	180
20750	199	20	40	60	80	100	119	139	159	179
20880	198	20	40	59	79	99	119	139	158	178
22000	197	20	39	59	79	99	118	138	158	177
22100	196	20	39	59	78	98	118	137	157	176
22200	195	20	39	59	78	98	117	137	156	176
22330	194	19	39	58	78	97	116	136	155	175
22440	193	19	39	58	77	97	116	135	154	174
22540	192	19	38	58	77	96	115	134	154	173
22680	191	19	38	57	76	96	115	134	153	172
22800	190	19	38	57	76	95	114	133	152	171
22900	189	19	38	57	76	95	113	132	151	170
23040	188	19	38	56	75	94	113	132	150	169

N.	0	1	2	3	4	5	6	7	8	9
231	636180	637999	639818	641636	643454	645270	647086	648902	650718	652534
232	654350	656167	657984	659802	661619	663436	665253	667070	668887	670704
233	672520	674337	676154	677971	679787	681603	683419	685235	687051	688867
234	690683	692499	694314	696130	697945	699760	701575	703390	705205	707020
235	708835	710649	712464	714278	716092	717906	719720	721534	723348	725162
236	726975	728789	730602	732415	734228	736041	737854	739667	741480	743293
237	745106	746918	748731	750543	752355	754167	755979	757791	759603	761415
238	763227	765038	766850	768661	770472	772284	774095	775906	777717	779528
239	781339	783150	784961	786772	788582	790393	792203	794014	795824	797634
240	799445	801255	803065	804875	806685	808495	810305	812114	813924	815733
241	817543	819352	821162	822971	824780	826589	828398	830207	832016	833825
242	835634	837443	839252	841060	842869	844677	846486	848294	850102	851910
243	853719	855527	857335	859143	860951	862759	864566	866374	868182	869989
244	871797	873604	875412	877219	879026	880833	882640	884447	886254	888061
245	889868	891675	893482	895289	897095	898902	900708	902515	904321	906128
246	907934	909740	911546	913352	915158	916964	918769	920575	922381	924186
247	925992	927797	929603	931408	933213	935018	936823	938628	940433	942238
248	944043	945847	947652	949456	951261	953065	954869	956674	958478	960282
249	962086	963890	965694	967498	969302	971106	972909	974713	976516	978320
250	980123	981926	983730	985533	987336	989139	990942	992745	994548	996351
251	998154	999957	001759	003562	005364	007167	008969	010771	012574	014376
252	016178	017980	019782	021584	023386	025188	026990	028791	030593	032394
253	034196	035997	037798	039600	041401	043202	045003	046804	048605	050406
254	052207	054008	055808	057609	059410	061210	063011	064811	066611	068412
255	070212	072012	073812	075612	077412	079212	081012	082811	084611	086411
256	088210	090010	091809	093608	095408	097207	099006	100805	102604	104403
257	106202	108001	109800	111599	113398	115196	116995	118793	120592	122390
258	124189	125987	127785	129583	131381	133179	134977	136775	138573	140371
259	142168	143966	145764	147561	149359	151156	152953	154751	156548	158345
260	160142	161939	163736	165533	167330	169127	170924	172721	174518	176314
261	178111	179907	181704	183500	185297	187093	188889	190686	192482	194278
262	196074	197870	199666	201462	203258	205054	206850	208645	210441	212237
263	214032	215828	217623	219418	221214	223009	224804	226599	228394	230189
264	231984	233779	235574	237369	239164	240958	242753	244547	246342	248136
265	249931	251725	253519	255313	257107	258901	260695	262489	264283	266077
266	267871	269664	271458	273252	275045	276839	278632	280425	282219	284012
267	285805	287598	289391	291184	292977	294770	296563	298356	300149	301941
268	303734	305527	307319	309112	310904	312697	314489	316281	318074	319866

PROPORTIONAL PARTS.

N.	D.	1	2	3	4	5	6	7	8	9
23150	187	19	37	56	75	94	112	131	150	168
23280	186	19	37	56	74	93	112	130	149	167
23400	185	19	37	56	74	93	111	130	148	167
23530	184	18	37	55	74	92	110	129	147	166
23650	183	18	37	55	73	92	110	128	146	165
23790	182	18	36	55	73	91	109	127	146	164
23950	181	18	36	54	72	91	109	127	145	163
24060	180	18	36	54	72	90	108	126	144	162
24200	179	18	36	54	72	90	107	125	143	161
24330	178	18	36	53	71	89	107	125	142	160
24470	177	18	35	53	71	89	106	124	142	159
24600	176	18	35	53	70	88	106	123	141	158
24740	175	18	35	53	70	88	105	123	140	158

N.	D.	1	2	3	4	5	6	7	8	9
24900	174	17	35	52	70	87	104	122	139	157
25040	173	17	35	52	69	87	104	121	138	156
25160	172	17	34	52	69	86	103	120	138	155
25320	171	17	34	51	68	86	103	120	137	154
25470	170	17	34	51	68	85	102	119	136	153
25680	169	17	34	51	68	85	101	118	135	152
25750	168	17	34	50	67	84	101	118	134	151
25950	167	17	33	50	67	84	100	117	134	150
26100	166	17	33	50	66	83	100	116	133	149
26250	165	17	33	50	66	83	99	116	132	149
26400	164	16	33	49	66	82	98	115	131	148
26550	163	16	33	49	65	82	98	114	130	147
26700	162	16	32	49	65	81	97	113	130	146

N.	0	1	2	3	4	5	6	7	8	9
269	4297523	299137	300751	302364	303976	305588	307199	308809	310419	312029
270	313638	315246	316853	318460	320067	321673	323278	324883	326487	328090
271	329693	331295	332897	334498	336098	337698	339296	340896	342495	344092
272	345689	347285	348881	350476	352071	353665	355258	356851	358444	360035
273	361626	363217	364807	366396	367985	369573	371160	372748	374334	375920
274	377506	379090	380675	382258	383841	385423	387005	388587	390167	391747
275	393327	394906	396484	398062	399639	401216	402792	404368	405943	407517
276	409091	410664	412237	413809	415380	416951	418522	420092	421661	423230
277	424798	426365	427932	429499	431065	432630	434195	435759	437322	438885
278	440448	442010	443571	445132	446692	448252	449811	451370	452928	454485
279	456042	457598	459154	460709	462264	463818	465373	466925	468477	470029
280	471580	473131	474681	476231	477780	479329	480877	482424	483971	485517
281	487063	488608	490153	491697	493241	494784	496327	497868	499410	500951
282	502491	504031	505570	507109	508647	510185	511722	513258	514794	516329
283	517864	519399	520932	522466	523998	525531	527062	528593	530124	531654
284	533183	534712	536211	537769	539296	540823	542349	543875	545400	546924
285	548449	549972	551495	553018	554540	556061	557582	559102	560622	562142
286	563660	565179	566696	568213	569730	571246	572762	574277	575791	577305
287	578819	580332	581844	583356	584868	586379	587889	589399	590908	592417
288	593925	595433	596940	598446	599953	601458	602963	604468	605972	607475
289	608978	610481	611983	613484	614985	616486	617986	619485	620984	622482
290	623980	625477	626974	628470	629966	631461	632956	634450	635944	637437
291	638930	640422	641914	643405	644895	646386	647875	649364	650853	652341
292	653829	655316	656802	658288	659774	661259	662743	664227	665711	667194
293	668676	670158	671640	673121	674601	676081	677561	679039	680518	681996
294	683473	684950	686427	687903	689378	690853	692327	693801	695275	696748
295	698220	699692	701164	702634	704105	705575	707044	708513	709982	711450
296	712917	714384	715851	717317	718782	720247	721711	723175	724639	726102
297	727564	729027	730488	731949	733410	734870	736329	737788	739247	740705
298	742163	743620	745076	746533	747988	749443	750898	752352	753806	755259
299	756712	758164	759616	761067	762518	763968	765418	766867	768316	769765
300	771213	772660	774107	775553	776999	778445	779890	781334	782778	784222
301	785665	787108	788550	789991	791432	792873	794313	795753	797192	798631
302	800069	801507	802945	804381	805818	807254	808689	810124	811559	812993
303	814426	815859	817292	818724	820156	821587	823018	824448	825878	827307
304	828736	830164	831592	833020	834446	835873	837299	838725	840150	841574
305	842998	844422	845845	847268	848690	850112	851533	852954	854375	855795
306	857214	858633	860052	861470	862888	864305	865722	867138	868554	869969
307	871384	872798	874212	875626	877039	878451	879863	881275	882686	884097
308	885507	886917	888326	889735	891144	892552	893959	895366	896773	898179
309	899585	900990	902395	903799	905203	906607	908010	909412	910814	912216

PROPORTIONAL PARTS.

N.	D.	1	2	3	4	5	6	7	8	9
26900	161	16	32	48	64	81	97	113	129	145
27050	160	16	32	48	64	80	96	112	128	144
27230	159	16	32	48	64	80	95	111	127	143
27400	158	16	32	47	63	79	95	111	126	142
27550	157	16	31	47	63	79	94	110	126	141
27750	156	16	31	47	62	78	94	109	125	140
27920	155	16	31	47	62	78	93	109	124	140
28100	151	15	31	46	62	77	92	108	123	139
28300	153	15	31	46	61	77	92	107	122	138
28500	152	15	30	46	61	76	91	106	122	137
28680	151	15	30	45	60	76	91	106	121	136

N.	D.	1	2	3	4	5	6	7	8	9
28850	150	15	30	45	60	75	90	105	120	135
29050	149	15	30	15	60	75	89	104	119	134
29250	148	15	30	44	59	74	89	104	118	133
29450	147	15	29	44	59	74	88	103	118	132
29650	146	15	29	44	58	73	88	102	117	131
29850	145	15	29	44	58	73	87	102	116	131
30050	144	14	29	43	58	72	86	101	115	130
30300	143	14	29	43	57	72	86	100	114	129
30500	142	14	28	43	57	71	85	99	114	128
30700	141	14	28	42	56	71	85	99	113	127
30900	140	14	28	42	56	70	84	98	112	126

N.	0	1	2	3	4	5	6	7	8	9
310	4913617	915019	916418	917818	919217	920616	922015	923413	924810	926207
311	927604	929000	930396	931791	933186	934581	935974	937368	938761	940154
312	941546	942938	944329	945720	947110	948500	949890	951279	952667	954056
313	955443	956831	958218	959604	960990	962375	963761	965145	966529	967913
314	969296	970679	972062	973444	974825	976206	977587	978967	980347	981727
315	983106	984484	985862	987240	988617	989994	991370	992746	994121	995496
316	996871	998245	999619							
	5			000992	002365	003737	005109	006481	007852	009222
317	010593	011962	013332	014701	016069	017437	018805	020172	021539	022905
318	024271	025637	027002	028366	029731	031094	032458	033821	035183	036545
319	037907	039268	040629	041989	043349	044709	046068	047426	048785	050142
320	051500	052857	054213	055569	056925	058280	059635	060990	062344	063697
321	065050	066403	067755	069107	070459	071810	073160	074511	075860	077210
322	078559	079907	081255	082603	083950	085297	086644	087990	089335	090680
323	092025	093370	094714	096057	097400	098743	100085	101427	102768	104109
324	105450	106790	108130	109469	110808	112147	113485	114823	116160	117497
325	118834	120170	121505	122841	124175	125510	126844	128178	129511	130841
326	132176	133508	134840	136171	137502	138832	140162	141491	142820	144149
327	145478	146805	148133	149460	150787	152113	153439	154764	156089	157414
328	158738	160062	161386	162709	164031	165354	166676	167997	169318	170639
329	171959	173279	174598	175917	177236	178554	179872	181189	182507	183823
330	185139	186455	187771	189086	190400	191715	193028	194342	195655	196968
331	198280	199592	200903	202214	203525	204835	206145	207455	208761	210073
332	211381	212689	213996	215303	216610	217916	219222	220528	221833	223138
333	224442	225746	227050	228353	229656	230958	232260	233562	234863	236164
334	237465	238765	240064	241364	242663	243961	245259	246557	247854	249151
335	250448	251744	253040	254336	255631	256925	258220	259513	260807	262100
336	263393	264685	265977	267269	268560	269851	271141	272431	273721	275010
337	276299	277588	278876	280163	281451	282738	284024	285311	286596	287882
338	289167	290452	291736	293020	294304	295587	296870	298152	299434	300716
339	301997	303278	304558	305838	307118	308398	309677	310955	312231	313512
340	314789	316066	317343	318619	319896	321171	322446	323721	324996	326270
341	327544	328817	330090	331363	332635	333907	335179	336450	337721	338991
342	340261	341531	342800	344069	345338	346606	347874	349141	350408	351675
343	352941	354207	355473	356738	358003	359267	360532	361795	363059	364322
344	365584	366847	368109	369370	370631	371892	373153	374113	375673	376932
345	378191	379450	380708	381966	383223	384481	385737	386994	388250	389506
346	390761	392016	393271	394525	395779	397032	398286	399538	400791	402043
347	403295	404546	405797	407048	408298	409548	410798	412047	413296	414544
348	415792	417040	418288	419535	420781	422028	423274	424519	425765	427010
349	428254	429498	430742	431986	433229	434472	435714	436956	438198	439439
350	440680	441921	443161	444401	445641	446880	448119	449358	450596	451834
351	453071	454308	455545	456781	458018	459253	460489	461724	462958	464193
352	465427	466660	467894	469126	470359	471591	472823	474055	475286	476517

PROPORTIONAL PARTS

N.	D.	1	2	3	4	5	6	7	8	9
31150	139	14	28	42	56	70	83	97	111	125
31350	138	14	28	41	55	69	83	97	110	124
31600	137	14	27	41	55	69	82	96	110	123
31800	136	14	27	41	54	68	82	95	109	122
32050	135	14	27	41	54	68	81	95	108	122
32300	134	13	27	40	54	67	80	94	107	121
32550	133	13	27	40	53	67	80	93	106	120
32750	132	13	26	40	53	66	79	92	106	119
33000	131	13	26	39	52	66	79	92	105	118

N	D.	1	2	3	4	5	6	7	8	9
33300	130	13	26	39	52	65	78	91	104	117
33550	129	13	26	39	52	65	77	90	103	116
33800	128	13	26	38	51	64	77	90	102	115
34050	127	13	25	38	51	64	76	89	102	114
34350	126	13	25	38	50	63	76	88	101	113
34600	125	13	25	38	50	63	75	88	100	113
34900	124	12	25	37	50	62	74	87	99	112
35160	123	12	25	37	49	62	74	86	98	111

N.	0	1	2	3	4	5	6	7	8	9
353	5477747	478977	480207	481436	482665	483894	485123	486351	487579	488806
354	490033	491259	492486	493712	494937	496162	497387	498612	499836	501060
355	502284	503507	504730	505952	507174	508396	509618	510839	512059	513280
356	514500	515720	516939	518158	519377	520595	521813	523031	524248	525465
357	526682	527899	529115	530330	531545	532760	533974	535189	536403	537617
358	538830	540043	541256	542468	543680	544892	546103	547314	548524	549735
359	550944	552154	553363	554572	555781	556989	558197	559404	560612	561818
360	563025	564231	565437	566643	567848	569053	570257	571461	572665	573869
361	575072	576275	577477	578680	579881	581083	582284	583485	584686	585886
362	587086	588285	589484	590683	591882	593080	594278	596476	796673	597870
363	599066	600262	601458	602654	603849	605044	606239	607433	608627	609821
364	611014	612207	613399	614592	615784	616975	618167	719358	620548	621739
365	622929	624118	625308	626497	627685	628874	630062	631250	632437	633624
366	634811	635997	637183	638369	639555	640740	641925	643109	644293	645477
367	646661	647844	649027	650209	651392	652573	653755	654936	656117	657298
368	658478	659658	660838	662017	663196	664375	665553	666731	667909	669087
369	670264	671140	672617	673793	674969	676144	677320	678495	679669	680843
370	682017	683191	684364	685537	686710	687882	689054	690226	691397	692568
371	693739	694910	696080	697249	698419	699588	700757	701926	703094	704262
372	705429	706597	707764	708930	710097	711263	712429	713594	714759	715924
373	717088	718252	719416	720580	721743	722906	724069	725231	726393	727555
374	728716	729877	731038	732198	733358	734518	735678	736837	737996	739154
375	740313	741471	742628	743786	744943	746099	747256	748412	749568	750723
376	751878	753033	754188	755342	756496	757650	758803	759956	761109	762261
377	763414	764565	765717	766868	768019	769170	770320	771470	772620	773769
378	774918	776067	777215	778363	779511	780659	781806	782953	784100	785246
379	786392	787538	688683	789828	790973	792118	793262	794406	795550	796693
380	797836	798979	800121	801263	802405	803547	804688	805829	806969	808110
381	809250	810389	811529	812668	813807	814945	816084	817222	818359	819497
382	820634	821770	822907	824013	825179	826314	827450	828585	829719	830854
383	831988	833122	834255	835388	836521	837654	838786	839918	841050	842181
384	843312	844443	845574	846704	847834	848963	850093	851222	852351	853479
385	854607	855735	856863	857990	859117	860244	861370	862496	863622	864748
386	865873	866998	868123	869247	870371	871495	872618	873742	874865	875987
387	877110	878232	879353	880475	881596	882717	883838	884958	886078	887198
388	888317	889436	890555	891674	892792	893910	895028	896145	897263	898379
389	899196	900612	901728	902844	903959	905075	906189	907304	908418	909532
390	910646	911760	912873	913986	915098	916210	917322	918434	919546	920657
391	921768	922878	923988	925098	926208	927318	928427	929536	930644	931753
392	932861	933968	935076	936183	967290	938397	939503	940609	941715	942820
393	943926	945030	946135	947239	948344	949447	950551	951654	952757	953860
394	954962	956064	957166	958268	959369	960170	961571	962671	963771	964871
395	965971	967070	968169	969268	970367	971465	972563	973661	974758	975855
396	976952	978048	979145	980241	981336	982432	983527	984622	985717	986811
397	987905	988999	990092	991186	992279	993371	994464	995556	996648	997739

PROPORTIONAL PARTS.

N.	D.	1	2	3	4	5	6	7	8	9
35950	122	12	24	37	49	61	73	85	98	110
35750	121	12	24	36	48	63	73	85	97	109
36050	120	12	24	36	48	60	72	84	96	108
36350	119	12	24	36	48	60	71	83	95	107
36650	118	12	24	35	47	59	71	83	94	106
36950	117	12	23	35	47	59	70	82	94	105
37250	116	12	23	35	46	58	70	81	93	101

N.	D.	1	2	3	4	5	6	7	8	9
37600	115	12	23	35	46	58	69	81	92	104
37950	114	11	23	34	46	57	68	80	91	103
38250	113	11	23	34	45	57	68	79	90	102
38550	112	11	22	34	45	56	67	78	90	101
38900	111	11	22	33	44	56	67	78	89	100
39300	110	11	22	33	44	55	66	77	88	99
39650	109	11	22	33	44	55	65	76	87	98

N.	0	1	2	3	4	5	6	7	8	9
398	5998631	999922								
	6		001013	002103	003193	004283	005373	006462	007551	008640
399	009729	010817	011906	012995	014084	015168	016255	017341	018428	019514
400	020600	021686	022773	023856	024941	026025	027109	028193	029277	030361
401	031444	032527	033610	034692	035774	036855	037937	039018	040099	041180
402	042261	043341	044421	045500	046580	047659	048738	049816	050895	051973
403	053050	054128	055205	056282	057359	058435	059512	060587	061663	062739
404	063814	064889	065963	067037	068111	069185	070259	071332	072405	073478
405	074550	075622	076694	077766	078837	079909	080979	082050	083120	084191
406	085260	086330	087399	088468	089537	090605	091674	092742	093809	094877
407	095944	097011	098078	099144	100210	101276	102342	103407	104472	105537
408	106602	107666	108730	109794	110857	111921	112984	114046	115109	116171
409	117233	118295	119356	120417	121478	122539	123599	124660	125720	126779
410	127839	128898	129957	131015	132074	133132	134189	135247	136304	137361
411	138418	139475	140531	141587	142643	143698	144754	145809	146863	147918
412	148972	150026	151080	152133	153187	154240	155292	156345	157397	158449
413	159501	160552	161603	162654	163705	164755	165805	166855	167905	168954
414	170003	171052	172101	173149	174197	175245	176293	177340	178387	179434
415	180481	181527	182573	183619	184665	185710	186755	187800	188845	189889
416	190933	191977	193021	194064	195107	196150	197193	198235	199277	200319
417	201361	202402	203443	204484	205524	206565	207605	208645	209684	210724
418	211763	212802	213840	214879	215917	216955	217992	219030	220067	221104
419	222140	223177	224213	225249	226284	227320	228355	229390	230424	231459
420	232493	233527	234560	235594	236627	237660	238693	239725	240757	241789
421	242821	243852	244884	245915	246945	247976	249006	250036	251066	252095
422	253125	254154	255182	256211	257239	258267	259295	260322	261350	262377
423	263404	264430	265457	266483	267509	268534	269560	270585	271610	272634
424	273659	274683	275707	276730	277754	278777	279800	280823	281845	282867
425	283889	284911	285933	286954	287975	288996	290016	291037	292057	293076
426	294096	295115	296134	297153	298172	299190	300209	301227	302245	303262
427	304279	305296	306312	307329	308345	309361	310377	311393	312408	313424
428	314438	315452	316467	317481	318495	319508	320522	321535	322548	323560
429	324573	325585	326597	327609	328620	329632	330643	331654	332664	333674
430	334685	335691	336701	337713	338723	339732	340740	341749	342757	343765
431	344773	345780	346788	347795	348801	349808	350814	351820	352826	353832
432	354837	355843	356848	357852	358857	359861	360865	361869	362873	363876
433	364879	365882	366884	367887	368889	369891	370893	371891	372895	373897
434	374897	375898	376898	377898	378897	379898	380897	381896	382895	383894
435	384893	385891	386889	387887	388884	389882	390879	391876	392872	393869
436	394865	395861	396857	397852	398847	399842	400837	401832	402826	403820
437	404814	405808	406802	407795	408788	409781	410773	411765	412758	413749
438	414741	415733	416724	417715	418705	419696	420686	421676	422666	423656
439	424643	425634	426623	427612	428601	429589	430577	431565	432552	433540
440	434527	435514	436500	437487	438473	439459	440445	441131	442416	443401
441	444386	445371	446355	447339	448323	449307	450291	451274	452257	453240
442	454223	455205	456187	457169	458151	459133	460114	461095	462076	463057
443	464037	465018	465998	466977	467957	468936	469915	470894	471873	472851

PROPORTIONAL PARTS.

N	D.	1	2	3	4	5	6	7	8	9
39650	109	11	22	33	44	55	65	76	87	98
40050	108	11	22	32	43	54	65	76	86	97
40450	107	11	21	32	43	54	64	75	86	96
40800	106	11	21	32	42	53	64	74	85	95
41200	105	11	21	32	42	53	63	74	84	95
41600	104	10	21	31	42	52	62	73	83	91

N.	D.	1	2	3	4	5	6	7	8	9
42000	103	10	21	31	41	52	62	72	82	93
42400	102	10	20	31	41	51	61	71	82	92
42800	101	10	20	30	40	51	61	71	81	91
43200	100	10	20	30	40	50	60	70	80	90
43600	99	10	20	30	40	50	59	69	79	89
44050	98	10	20	29	39	49	59	69	78	88

N.	0	1	2	3	4	5	6	7	8	9
444	6473830	474808	475786	476763	477741	478718	479695	480671	481648	482624
445	483600	484576	485552	486527	487502	488477	489452	490426	491401	492375
446	493349	494322	495296	496269	497242	498215	499187	500160	501132	502104
447	503075	504047	505018	505989	506960	507930	508901	509871	510841	511811
448	512780	513749	514719	515687	516656	517624	518593	519561	520528	521496
449	522463	523431	524397	525364	526331	527297	528264	529229	530195	531160
450	532125	533090	534055	535019	535984	536948	537912	538876	539839	540802
451	541765	542728	543691	544653	545616	546578	547539	548501	549462	550423
452	551384	552345	553306	554266	555226	556186	557145	558105	559064	560023
453	560982	561941	562899	563857	564815	565773	566730	567688	568645	569602
454	570559	571515	572471	573427	574383	575339	576294	577250	578205	579159
455	580114	581068	582023	582977	583930	584884	585837	586790	587743	588696
456	589648	590601	591553	592505	593456	594408	595358	596310	597261	598212
457	599162	600112	601062	602012	602962	603911	604860	605809	606758	607706
458	608655	609603	610551	611499	612446	613393	614341	615287	616234	617181
459	618127	619073	620019	620964	621910	622855	623800	624745	625690	626634
460	627578	628522	629466	630410	631353	632296	633239	634182	635125	636067
461	637009	637951	638893	639835	640776	641717	642658	643599	644539	645480
462	646420	647360	648299	649239	650178	651117	652056	652995	653934	654872
463	655810	656748	657686	658623	659560	660497	661434	662371	663307	664244
464	665180	666116	667051	667987	668922	669857	670792	671727	672661	673595
465	674530	675463	676397	677331	678264	679197	680130	681062	681995	682927
466	683859	684791	685723	686654	687585	688516	689447	690378	691308	692239
467	693169	694099	695028	695958	696887	697816	698745	699674	700602	701530
468	702459	703386	704314	705242	706169	707096	708023	708950	709876	710802
469	711728	712654	713580	714506	715431	716356	717281	718206	719130	720054
470	720979	721903	722826	723750	724673	725596	726519	727442	728365	729287
471	730209	731131	732053	732974	733896	734817	735738	736659	737579	738500
472	739420	740340	741260	742179	743099	744018	744937	745856	746775	747693
473	748611	749529	750447	751365	752283	753200	754117	755034	755951	756867
474	757783	758700	759615	760531	761447	762362	763277	764192	765107	766022
475	766936	767850	768764	769678	770592	771505	772418	773332	774244	775157
476	776070	776982	777894	778806	779718	780629	781540	782452	783362	784273
477	785184	786094	787004	787914	788824	789734	790643	791552	792461	793370
478	794279	795187	796096	797004	797912	798819	799727	800634	801541	802448
479	803355	804262	805168	806074	806980	807886	808792	809697	810602	811507
480	812412	813317	814222	815126	816031	816934	817838	818741	819645	820548
481	821451	822354	823256	824159	825061	825963	826865	827766	828668	829569
482	830470	831371	832272	833173	834073	834973	835873	836773	837673	838572
483	839471	840370	841269	842168	843066	843965	844863	845761	846659	847556
484	848454	849351	850248	851145	852041	852938	853834	854730	855626	856522
485	857417	858313	859208	860103	860998	861892	862787	863681	864575	865469
486	866363	867256	868150	869043	869936	870828	871721	872613	873506	874398
487	875290	876181	877073	877964	878855	879746	880637	881528	882418	883308
488	884198	885088	885978	886867	887757	888646	889535	890423	891312	892200
489	893089	893977	894864	895752	896640	897527	898414	899301	900188	901074
490	901961	902847	903733	904619	905505	906390	907275	908161	909046	909930

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
44500	97	10	19	29	39	49	58	68	78	87
45000	96	10	19	29	38	48	58	67	77	86
45500	95	10	19	29	38	48	57	67	76	86
46000	94	9	19	28	38	47	56	66	75	85
46500	93	9	19	28	37	47	56	65	74	84

N.	D	1	2	3	4	5	6	7	8	
47000	92	9	18	28	37	46	55	64	74	83
47500	91	9	18	27	36	46	55	64	73	82
48000	90	9	18	27	36	45	54	63	72	81
48500	89	9	18	27	36	45	53	62	71	80
49100	88	9	18	26	35	44	53	62	70	79

N.	0	1	2	3	4	5	6	7	8	9
491	6910815	911699	912583	913468	914352	915235	916119	917002	917885	918768
492	919651	920534	921418	922299	923180	924062	924944	925826	926707	927588
493	928469	929350	930231	931111	931991	932872	933752	934631	935511	936390
494	937269	938149	939027	939906	940785	941663	942541	943419	944297	945175
495	946052	946929	947806	948683	949560	950437	951313	952189	953065	953941
496	954817	955692	956567	957443	958318	959193	960067	960942	961816	962690
497	963564	964438	965313	966185	967058	967931	968804	969676	970549	971421
498	972293	973165	974037	974909	975780	976652	977523	978394	979264	980135
499	981005	981876	982746	983616	984485	985355	986224	987093	987963	988831
500	989700	990569	991437	992305	993173	994041	994908	995776	996643	997510
501	998377	999244								
	7		000111	000977	001843	002709	003575	004441	005307	006172
502	007037	007902	008767	009632	010496	011361	012225	013089	013953	014816
503	015680	016543	017406	018269	019132	019995	020857	021720	022582	023444
504	024305	025167	026028	026890	027751	028612	029472	030333	031193	032054
505	032914	033774	034633	035493	036352	037212	038071	038930	039788	040647
506	041505	042363	043221	044079	044937	045794	046652	047509	048366	049223
507	050080	050936	051793	052649	053505	054360	055216	056072	056927	057782
508	058637	059492	060347	061201	062055	062910	063764	064617	065471	066325
509	067178	068031	068884	069737	070589	071442	072294	073146	073998	074850
510	075702	076553	077405	078256	079107	079957	080808	081659	082509	083359
511	084209	085059	085908	086758	087607	088456	089305	090154	091003	091851
512	092700	093548	094396	095244	096091	096939	097786	098633	099480	100327
513	101174	102020	102866	103713	104559	105404	106250	107096	107941	108786
514	109631	110476	111321	112165	113010	113854	114698	115542	116385	117229
515	118072	118915	119759	120601	121444	122287	123129	123971	124813	125655
516	126497	127339	128180	129021	129862	130703	131544	132385	133225	134065
517	134905	135745	136585	137425	138264	139104	139943	140782	141620	142459
518	143298	144136	144974	145812	146650	147488	148325	149162	150000	150837
519	151674	152510	153347	154183	155019	155856	156691	157527	158363	159198
520	160033	160869	161703	162538	163373	164207	165042	165876	166710	167541
521	168377	169211	170044	170877	171710	172543	173376	174208	175041	175873
522	176705	177537	178369	179200	180032	180863	181694	182525	183356	184186
523	185017	185847	186677	187507	188337	189167	189996	190826	191655	192484
524	193313	194142	194970	195799	196627	197455	198283	199111	199938	200766
525	201593	202420	203247	204074	204901	205727	206554	207380	208206	209032
526	209857	210683	211508	212334	213159	213984	214809	215633	216458	217282
527	218106	218930	219754	220578	221401	222225	223048	223871	224694	225517
528	226339	227162	227984	228806	229628	230450	231272	232093	232914	233736
529	234557	235378	236198	237019	237839	238660	239480	240300	241120	241939
530	242759	243576	244397	245216	246035	246854	247672	248491	249309	250127
531	250945	251763	252581	253398	254216	255033	255850	256667	257483	258300
532	259116	259933	260749	261565	262380	263196	264012	264827	265642	266457
533	267272	268087	268901	269716	270530	271344	272158	272972	273786	274599
534	275413	276226	277039	277852	278664	279477	280290	281102	281914	282726
535	283538	284350	285161	285972	286784	287595	288406	289216	290027	290838
536	291648	292458	293268	294078	294888	295697	296507	297316	298125	298934
537	299743	300552	301360	302168	302977	303785	304593	305400	306208	307015
538	307823	308630	309437	310244	311051	311857	312663	313470	314276	315082

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
49700	87	9	17	26	35	44	52	61	70	78
50250	86	9	17	26	34	43	52	60	69	77
50800	85	9	17	26	34	43	51	60	68	77
51400	84	8	17	25	34	42	50	59	67	76

N.	D	1	2	3	4	5	6	7	8	9
52000	83	8	17	25	33	42	50	58	66	75
52600	82	8	16	25	33	41	49	57	66	74
53200	81	8	16	24	32	41	49	57	65	73
53900	80	8	16	24	32	40	48	56	64	72

N.	0	1	2	3	4	5	6	7	8	9
539	7315888	316693	317499	318304	319109	319914	[illegible]	321524	322329	323133
540	323938	324742	325546	327350	327153	327957	[illegible]	329564	330367	331170
541	331973	332775	333578	334380	335183	335985	[illegible]	337588	338390	339192
542	339993	340794	341595	342396	343197	343997	[illegible]	345598	346398	347198
543	347998	348798	349598	350397	351196	351995	[illegible]	353593	354392	355191
544	355989	356787	357585	358383	359181	359979	[illegible]	361574	362371	363168
545	363965	364762	365558	366355	367151	367946	[illegible]	369540	370335	371131
546	371926	372722	373517	374312	375107	375902	[illegible]	377491	378285	379079
547	379873	380667	381461	382254	383048	383841	[illegible]	385427	386220	387013
548	387806	388598	389390	390182	390974	391766	[illegible]	393350	394141	394932
549	395723	396514	397305	398096	398887	399677	[illegible]	401257	402047	402837
550	403627	404416	405206	405995	406784	407573	[illegible]	409151	409939	410728
551	411516	412304	413092	413880	414668	415455	[illegible]	417030	417817	418604
552	419391	420177	420964	421750	422537	423323	[illegible]	424895	425680	426466
553	427251	428037	428822	429607	430392	431176	[illegible]	432745	433530	434314
554	435098	435882	436665	437449	438232	439016	[illegible]	440582	441365	442147
555	442930	443712	444495	445277	446059	446841	[illegible]	448404	449185	449967
556	450748	451529	452310	453091	453871	454652	[illegible]	456212	456992	457772
557	458552	459332	460111	460890	461670	462449	[illegible]	464006	464785	465564
558	466342	467120	467898	468676	469454	470232	471009	471787	472564	473341
559	474118	474895	475672	476448	477225	478001	478777	479553	480329	481105
560	481880	482656	483431	484206	484981	485756	486531	487306	488080	488854
561	489629	490403	491177	491950	492724	493498	494271	495044	495817	496590
562	497363	498136	498908	499681	500453	501225	501997	502769	503541	504312
563	505084	505855	506626	507398	508168	508939	509710	510480	511251	512021
564	512791	513561	514331	515101	515870	516639	517409	518178	518947	519716
565	520484	521253	522022	522790	523558	524326	525094	525862	526629	527397
566	528164	528932	529699	530466	531232	531999	532766	533532	534298	535065
567	535831	536596	537362	538128	538893	539659	540424	541189	541954	542719
569	543483	544248	545012	545777	546541	547305	548069	548832	549596	550359
569	551123	551886	552649	553412	554175	554937	555700	556462	557224	557987
570	558749	559510	560272	561034	561795	562556	563318	564079	564840	565600
571	566361	567122	567882	568642	569402	570162	570922	571682	572442	573201
572	573960	574719	575479	576237	576996	577755	578513	579272	580030	580788
573	581546	582304	583062	583819	584577	585334	586091	586848	587605	588362
574	589119	589875	590632	591388	592144	592900	593656	594412	595168	595923
575	596678	597431	598189	598944	599699	600453	601208	601962	602717	603471
576	604225	604979	605733	606486	607240	606993	608746	609500	610253	611005
577	611758	612511	613263	614016	614768	615520	616272	617024	617775	618527
578	619278	620030	620781	621532	622283	623034	623784	624535	625285	626035
579	626786	627536	628286	629035	629785	630534	631284	632033	632782	633531
580	634280	635029	635777	636526	637274	638022	638770	639518	640266	641014
581	641761	642509	643256	644003	644750	645497	646244	646991	647737	648484
582	649230	649976	650722	651468	652214	652959	653705	654450	655195	655941
56	656686	657430	658175	658920	659664	660409	661153	661897	662641	663385
584	664128	664872	665616	666359	667102	667845	668588	669331	670074	670816
585	671559	672301	673043	673785	674527	675269	676011	676752	677494	678235
586	678976	679717	680458	681199	681940	682680	683421	684161	684901	68564

PROPORTIONAL PARTS.

N	D	1	2	3	4	5	6	7	8	9
5160	77	8	16	23	32	40	47	5	63	71
55400	78	8	16	23	31	39	47	55	62	70
55000	77	8	15	23	31	39	46	54	62	69

N.	D	1	2	3	4	5	6	7	8	9
56700	76	8	15	23	30	38	46	53	61	68
57500	75	8	15	23	30	38	45	53	60	68
58300	74	7	15	22	30	37	44	52	59	67

N.	0	1	2	3	4	5	6	7	8	9
587	686381	687121	687860	688600	689339	690079	690818	691557	692296	693035
588	693773	694512	695250	695988	696727	697465	698203	698940	699678	700416
589	701153	701890	702627	703364	704101	704838	705575	706311	707048	707784
590	708520	709256	709992	710728	711463	712199	712934	713670	714405	715140
591	715875	716610	717344	718079	718813	719547	720282	721016	721750	722483
592	723217	723951	724684	725417	726150	726884	727616	728349	729082	729815
593	730547	731279	732011	732743	733475	734207	734939	735670	736402	737133
594	737864	738596	739326	740057	740788	741519	742249	742979	743710	744440
595	745170	745900	746629	747359	748088	748818	749547	750276	751005	751734
596	752463	753191	753920	754648	755376	756104	756832	757560	758288	759016
597	759743	760471	761198	761925	762652	763379	764106	764833	765559	766286
598	767012	767738	768464	769190	769916	770642	771367	772093	772818	773543
599	774268	774993	775718	776443	777167	777892	778616	779340	780065	780789
600	781513	782236	782960	783683	784407	785130	785853	786576	787299	788022
601	788745	789467	790190	790912	791634	792356	793078	793800	794522	795243
602	795965	796686	797408	798129	798850	799571	800291	801012	801732	802453
603	803173	803893	804613	805333	806053	806773	807492	808212	808931	809650
604	810369	811088	811807	812526	813245	813963	814681	815400	816118	816836
605	817554	818272	818989	819707	820424	821141	821859	822576	823293	824010
606	824726	825443	826159	826876	827592	828308	829024	829740	830456	831171
607	831887	832602	833318	834033	834748	835463	836178	836892	837607	838321
608	839036	839750	840464	841178	841892	842606	843319	844033	844746	845460
609	846173	846886	847599	848312	849024	849737	850450	851162	851874	852586
610	853298	854010	854722	855434	856145	856857	857568	858279	858990	859701
611	860412	861123	861833	862544	863254	863965	864675	865385	866095	866805
612	867514	868224	868933	869643	870352	871061	871770	872479	873188	873896
613	874605	875313	876021	876730	877438	878146	878854	879561	880269	880976
614	881684	882391	883098	883805	884512	885219	885926	886632	887339	888045
615	888751	889457	890163	890869	891575	892281	892986	893692	894397	895102
616	895807	896512	897217	897922	898626	899331	900035	900739	901441	902148
617	902852	903555	904259	904963	905666	906370	907073	907776	908479	909182
618	909885	910587	911290	911992	912695	913297	914099	914801	915503	916205
619	916906	917608	918309	919011	919712	920113	921114	921815	922516	923216
620	923917	924617	925318	926018	926718	927418	928118	928817	929517	930217
621	930916	931615	932314	933014	933712	934411	935110	935809	936507	937206
622	937904	938602	939300	939998	940696	941394	942091	942789	943486	944183
623	944880	945578	946274	946971	947668	948365	949061	949757	950454	951150
624	951846	952542	953238	953933	954629	955324	956020	956715	957410	958105
625	958800	959495	960190	960884	961579	962273	962967	963662	964356	965050
626	965743	966437	967131	967824	968517	969211	969904	970597	971290	971983
627	972675	973368	974060	974753	975445	976137	976829	977521	978213	978905
628	979596	980288	980979	981671	982362	983053	983744	984435	985125	985816
629	986506	987197	987887	988577	989267	989957	990647	991337	992027	992716
630	993405	994095	994784	995473	996162	996851	997540	998228	998917	999605
631	000294	000982	001670	002358	003046	0037 4	004421	005109	005796	006484
632	007171	007858	008545	009232	009919	010605	011292	011978	012665	013351
633	014037	014723	015409	016095	016781	017466	018152	018837	019523	020208
634	020893	021578	022262	022947	023632	024316	025001	025685	026369	027053

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
59100	73	7	15	22	29	37	44	51	58	66
59900	72	7	14	22	28	36	43	50	58	65
60700	71	7	14	21	28	36	43	50	57	64

N.	D	1	2	3	4	5	6	7	8	9
61550	70	7	14	21	28	35	42	49	56	63
62400	69	7	14	21	28	35	41	48	55	62
63250	68	7	14	20	27	34	41	48	54	61

N.	0	1	2	3	4	5	6	7	8	9
635	027737	028421	029105	029789	030472	031156	[illegible]	032522	033205	033888
636	034571	035254	035937	036619	037302	037984	[illegible]	039348	040031	040712
637	041394	042076	042758	043439	044121	044803	[illegible]	046164	046845	047526
638	048207	048887	049568	050248	050929	051609	[illegible]	052969	053649	054329
639	055009	055688	056368	057047	057726	058405	[illegible]	059764	060442	061121
640	061800	062478	063157	063835	064513	065191	[illegible]	066547	067225	067903
641	068580	069258	069935	070612	071290	071967	[illegible]	073320	073997	074674
642	075350	076027	076703	077379	078055	078731	[illegible]	080083	080759	081434
643	082110	082785	083460	084136	084811	085486	[illegible]	086835	087510	088184
644	088859	089533	090207	090881	091555	092229	[illegible]	093577	094250	094924
645	095597	096270	096944	097617	098290	098962	[illegible]	100308	100980	101653
646	102325	102997	103670	104342	105013	105685	[illegible]	107029	107700	108372
647	109043	109714	110385	111056	111727	112398	[illegible]	113739	114409	115080
648	115750	116420	117090	117760	118430	119100	[illegible]	120439	121108	121778
649	122447	123116	123785	124454	125123	125792	[illegible]	127129	127797	128465
650	129134	129802	130470	131138	131805	132473	[illegible]	133808	134475	135143
651	135810	136477	137144	137811	138478	139144	[illegible]	140477	141144	141810
652	142476	143142	143808	144474	145140	145805	[illegible]	147136	147801	148467
653	149132	149797	150462	151127	151791	152456	[illegible]	153785	154449	155113
654	155777	156441	157105	157769	158433	159097	[illegible]	160423	161087	161750
655	162413	163076	163739	164402	165064	165727	[illegible]	167052	167714	168376
656	169038	169700	170362	171024	171686	172347	[illegible]	173670	174331	174993
657	175654	176315	176976	177636	178297	178958	[illegible]	180278	180939	181599
658	182259	182919	183579	184239	184898	185558	[illegible]	186877	187536	188195
659	188854	189513	190172	190831	191489	192148	[illegible]	193465	194123	194781
660	195439	196097	196755	197413	198071	198728	[illegible]	200043	200700	201358
661	202015	202672	203328	203985	204642	205298	[illegible]	206611	207268	207924
662	208580	209286	209892	210548	211203	211859	[illegible]	213170	213825	214480
663	215135	215790	216445	217100	217755	218409	[illegible]	219718	220372	221027
664	221681	222335	222989	223643	224296	224950	[illegible]	226257	226910	227563
665	228216	228869	229522	230175	230828	231481	[illegible]	232786	233438	234090
666	23474	235394	236046	236698	237350	238002	[illegible]	239305	239956	240607
667	241258	241909	242560	243211	243862	244513	[illegible]	245814	246464	247114
668	247765	248415	249065	249715	250364	251014	[illegible]	252313	252963	253612
669	254261	254910	255559	256208	256857	257506	[illegible]	258803	259451	260100
670	260748	261396	262044	262692	263340	263988	[illegible]	265283	265931	266578
671	267225	267872	268519	269166	269813	270460	[illegible]	271753	272400	273046
672	273693	274339	274985	275631	276277	276923	[illegible]	278214	278860	279505
673	280151	280796	281441	282086	282731	283376	[illegible]	284665	285310	285955
674	286599	287243	287877	288532	289176	289820	[illegible]	291107	291751	292394
675	293038	293681	294324	294967	295611	296254	[illegible]	297539	298182	298824
676	299467	300109	300752	301394	302036	302678	[illegible]	303962	304604	305245
677	305887	306528	307169	307811	308452	309093	[illegible]	310375	311016	311656
678	312297	312937	313578	314218	314858	315499	[illegible]	316778	317418	318058
679	318698	319337	319977	320616	321255	321895	[illegible]	323173	323812	324450
680	325089	325728	326366	327005	327643	328281	[illegible]	329558	330195	330833
681	331471	332109	332746	333384	334021	334659	[illegible]	335933	336570	337207
682	337844	338480	339117	339754	340390	341027	[illegible]	342299	342935	343571

PROPQRTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
63400	68	7	14	20	27	34	41	48	54	61
64350	67	7	13	20	27	34	40	47	54	60
65300	66	7	13	20	26	33	40	46	53	59

N.	D	1	2	3	4	5	6	7	8	9
66250	65	7	13	20	26	33	39	46	52	59
67300	64	6	13	19	26	32	38	45	51	58
68400	63	6	13	19	25	32	38	44	50	57

N.	0	1	2	3	4	5	6	7	8	9
683	344807	344843	344879	344114	344750	347385	344021	348656	349291	349926
684	350561	351196	351831	352465	353100	353735	354369	355003	355638	356272
685	356906	357540	358174	358807	359441	360075	360708	361341	361975	362608
686	363241	363874	364507	365140	365773	366405	367038	367670	368303	368935
687	369567	370199	370832	371463	372095	372727	373359	373990	374622	375253
688	375884	376516	377147	377778	378409	379039	379670	380301	380931	381562
689	382192	382822	383452	384083	384713	385343	385973	386602	387232	387861
690	388491	389120	389750	390379	391008	391637	392266	392895	393523	394152
691	394780	395409	396037	396666	397294	397922	398550	399178	399806	400433
692	401061	401688	402316	402943	403571	404198	404825	405452	406079	406706
693	407332	407959	408586	409212	409838	410465	411091	411717	412343	412969
694	413595	414220	414846	415472	416097	416723	417348	417973	418598	419223
695	419848	420473	421098	421722	422347	422971	423596	424220	424844	425468
696	426092	426716	427340	427964	428588	429211	429835	430458	431081	431705
697	432328	432951	433574	434197	434819	435442	436065	436687	437310	437932
698	438554	439176	439799	440420	441042	441664	442286	442907	443529	444150
699	444772	445393	446014	446635	447256	447877	448498	449119	449739	450360
700	450980	451601	452221	452841	453461	454081	454701	455321	455941	456561
701	457180	457800	458419	459038	459658	460277	460896	461515	462134	462752
702	463371	463990	464608	465227	465845	466464	467081	467700	468318	468935
703	469553	470171	470789	471406	472024	472641	473258	473876	474493	475110
704	475727	476343	476960	477577	478193	478810	479126	480043	480659	481275
705	481891	482507	483123	483739	484355	484970	485586	486201	486817	487432
706	488047	488662	489277	489892	490507	491122	491736	492351	492965	493580
707	494194	494808	495423	496037	496651	497265	497878	498492	499106	499719
708	500333	500946	501559	502172	502786	503399	504011	504624	505237	505850
709	506462	507075	507687	508300	508912	509524	510136	510748	511360	511972
710	512583	513195	513807	514418	515030	515641	516252	516863	517474	518085
711	518696	519307	519917	520528	521139	521749	522359	522970	523580	524190
712	524800	525410	526020	526629	527239	527849	528458	529068	529677	530286
713	530895	531504	532113	532722	533331	533940	534546	535157	535765	536374
714	536982	537590	538198	538807	539414	540022	540630	541236	541845	542453
715	543060	543668	544275	544882	545489	546096	546703	547310	547917	548524
716	549130	549737	550343	550950	551556	552162	552768	553374	553980	554586
717	555192	555797	556403	557008	557614	558219	558824	559429	560035	560640
718	561244	561849	562454	563059	563663	564268	564872	565476	566081	566685
719	567289	567893	568197	569101	569704	570308	570912	571515	572118	572722
720	573325	573928	574531	575134	575737	576340	576943	577545	578148	578750
721	579353	579955	580557	581159	581761	582363	582965	583567	584169	584770
722	585372	585973	586575	587176	587777	588379	588980	589581	590181	590782
723	591383	591984	592584	593185	593785	594385	594986	595586	596186	596786
724	597386	597985	598585	599185	599784	600384	600983	601583	602182	602781
725	603380	603979	604578	605177	605776	606374	606973	607571	608170	608768
726	609366	609964	610562	611160	611758	612356	612954	613552	614149	614747
727	615344	615941	616539	617136	617733	618330	618927	619524	620121	620717
728	621314	621910	622507	623103	623699	624296	624892	625488	626084	626680
729	627275	627871	628467	629062	629658	630253	630848	631443	632039	632634
730	633229	633823	634418	635013	635608	636202	636797	637391	637985	638580
731	639174	639768	640362	640956	641550	642143	642737	643331	643924	644517

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
69500	62	6	12	19	25	31	37	43	50	56
70600	61	6	12	18	24	31	37	43	49	55

N.	D	1	2	3	4	5	6	7	8	9
71800	60	6	12	18	24	30	36	42	48	54
73000	59	6	12	18	24	30	35	41	47	53

N.	0	1	2	3	4	5	6	7	8	9
732	645111	645704	646297	646890	647483	648076	648668	649262	649855	650447
733	651040	651632	652225	652817	653409	654001	654593	655185	655777	656369
734	656961	657553	658144	658735	659327	659918	660509	661100	661691	662282
735	662873	663464	664055	664646	665236	665827	666417	667008	667598	668188
736	668778	669368	669958	670548	671138	671728	672317	672907	673496	674086
737	674675	675264	675853	676442	677031	677620	678209	678798	679387	679975
738	680564	681152	681740	682329	682917	683505	684093	684681	685269	685857
739	686444	687032	687620	688207	688794	689381	689968	690556	691143	691730
740	692317	692904	693491	694077	694664	695251	695837	696424	697010	697596
741	698182	698768	699354	699940	700526	701112	701697	702283	702868	703454
742	704039	704624	705210	705795	706380	706965	707549	708134	708719	709304
743	709888	710473	711057	711641	712226	712810	713394	713978	714562	715146
744	715729	716313	716897	717480	718064	718647	719230	719814	720397	720980
745	721563	722146	722728	723311	723894	724476	725059	725641	726224	726806
746	727388	727970	728552	729134	729716	730298	730880	731462	732043	732625
747	733206	733787	734369	734950	735531	736112	736693	737274	737855	738435
748	739016	739597	740177	740757	741338	741918	742498	743078	743658	744238
749	744818	745398	745978	746557	747137	747716	748296	748875	749454	750034
750	750613	751192	751771	752349	752928	753507	754086	754664	755243	755821
751	756399	756978	757556	758134	758712	759290	759868	760446	761023	761601
752	762178	762756	763333	763911	764488	765065	765642	766219	766796	767373
753	767950	768526	769103	769680	770256	770833	771409	771985	772561	773137
754	773713	774289	774865	775441	776017	776592	777168	777743	778319	778894
755	779470	780045	780620	781195	781770	782345	782919	783494	784069	784643
756	785218	785792	786367	786941	787515	788089	788663	789237	789811	790385
757	790959	791532	792106	792680	793253	793826	794400	794973	795546	796119
758	796692	797265	797838	798411	798983	799556	800128	800701	801273	801846
759	802418	802990	803562	804134	804706	805278	805850	806421	806993	807564
760	808136	808707	809279	809850	810421	810992	811563	812134	812705	813276
761	813847	814417	814988	815558	816129	816699	817269	817840	818410	818980
762	819550	820120	820689	821259	821829	822398	822968	823537	824107	824676
763	825245	825815	826384	826953	827522	828090	828659	829228	829797	830365
764	830934	831502	832070	832639	833207	833775	834343	834911	835479	836047
765	836614	837182	837750	838317	838885	839452	840019	840586	841154	841721
766	842288	842855	843421	843988	844555	845122	845688	846255	846821	847387
767	847954	848520	849086	849652	850218	850784	851350	851915	852481	853047
768	853612	854178	854743	855308	855874	856439	857004	857569	858134	858699
769	859263	859828	860393	860957	861522	862086	862651	863215	863779	864343
770	864907	865471	866035	866599	867163	867726	868290	868854	869417	869980
771	870541	871107	871670	872233	872796	873359	873922	874485	875048	875610
772	876173	876736	877298	877860	878423	878985	879547	880109	880671	881233
773	881795	882357	882918	883480	884042	884603	885165	885726	886287	886848
774	887410	887971	888532	889093	889653	890214	890775	891336	891896	892457
775	893017	893577	894138	894698	895258	895818	896378	896938	897498	898058
776	898617	899177	899736	900296	900855	901415	901974	902533	903092	903651
777	904210	904769	905328	905887	906445	907004	907563	908121	908679	909238
778	909796	910354	910912	911470	912028	912586	913144	913702	914259	914817
779	915375	915932	916489	917047	917604	918161	918718	919275	919832	920389
780	920946	921503	922059	922616	923173	923729	924285	924842	925398	925954

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
74250	56	6	12	17	23	29	35	41	46	52
75520	57	6	11	17	23	29	34	40	46	51

N.	D	1	2	3	4	5	6	7	8	9
76860	56	6	11	17	22	28	34	39	45	50
78250	55	6	11	17	22	28	33	39	44	50

N.	0	1	2	3	4	5	6	7	8	9
781	926510	927066	927622	928178	928734	929290	929846	930401	930957	931512
782	932068	932623	933179	933733	934288	934843	935398	935953	936508	937063
783	937618	938172	938727	939281	939836	940390	940944	941499	942053	942607
784	943161	943715	944269	944822	945376	945929	946483	947037	947590	948143
785	948697	949250	949803	950356	950909	951462	952015	952568	953120	953673
786	954225	954778	955330	955883	956435	956987	957539	958092	958644	959195
787	959748	960299	960851	961403	961954	962506	963057	963608	964160	964711
788	965262	965813	966364	966915	967466	968017	968568	969118	969669	970220
789	970770	971320	971871	972421	972971	973521	974071	974621	975171	975721
790	976271	976821	977370	977920	978469	979019	979568	980117	980667	981216
791	981765	982314	982863	983412	983960	984509	985058	985606	986155	986703
792	987252	987800	988348	988897	989445	989993	990541	991089	991636	992184
793	992732	993279	993827	994375	994922	995469	996017	996564	997111	997658
794	998205	998752	999299	999846						
					000392	000939	001486	002032	002579	003125
795	003671	004218	004764	005310	005856	006402	006948	007494	008039	008585
796	009131	009676	010222	010767	011313	011858	012403	012948	013493	014038
797	014583	015128	015673	016218	016762	017307	017851	018396	018940	019485
798	020029	020573	021117	021661	022205	022749	023293	023837	024381	024924
799	025468	026011	026555	027098	027641	028185	028728	029271	029814	030357
800	030900	031443	031985	032528	033071	033613	034156	034698	035241	035783
801	036325	036867	037409	037951	038493	039035	039577	040119	040661	041202
802	041744	042285	042827	043368	043909	044450	044992	045533	046074	046615
803	047155	047696	048237	048778	049318	049859	050399	050940	051480	052020
804	052560	053101	053641	054181	054721	055260	055800	056340	056880	057419
805	057959	058498	059038	059577	060116	060655	061195	061734	062273	062812
806	063350	063889	064428	064967	065505	066044	066582	067121	067659	068197
807	068735	069273	069812	070350	070887	071425	071963	072501	073038	073576
808	071114	071651	075188	075726	076263	076800	077337	077874	078411	078948
809	079485	080022	080559	081095	081632	082169	082705	083241	083778	084314
810	084850	085386	085922	086458	086994	087530	088066	088602	089137	089673
811	090209	090744	091279	091815	092350	092885	093420	093955	094490	095025
812	095560	096095	096630	097165	097699	098234	098768	099303	099837	100371
813	100905	101440	101974	102508	103042	103576	104109	104643	105177	105710
814	106244	106778	107311	107844	108378	108911	109411	109977	110510	111013
815	111576	112109	112642	113174	113707	114240	114772	115305	115837	116369
816	116902	117434	117966	118498	119030	119562	120094	120626	121157	121689
817	122221	122752	123284	123815	124346	124878	125409	125940	126471	127002
818	127533	128064	128595	129126	129656	130187	130717	131248	131778	132309
819	132839	133369	133899	134430	134960	135490	136019	136549	370791	137609
820	138139	138668	139198	139727	140257	140786	141315	141844	142373	142903
821	143432	143961	144489	145018	145547	146076	146604	147133	147661	148190
822	148718	149246	149775	150303	150831	151359	151887	152415	152943	153471
823	153998	154526	155054	155581	156109	156636	157163	157691	158218	158745
824	159272	159799	160326	160853	161380	161907	162433	162960	163487	164013
825	164539	165066	165592	166118	166645	167171	167697	168223	168749	169275
826	169800	170326	170852	171378	171903	172429	172954	173479	174005	174530
827	175055	175580	176105	176630	177155	177680	178205	178730	179254	179779
828	180303	180828	181352	181877	182401	182925	183449	183973	184497	185021
829	185545	186069	186593	187117	187640	188164	188687	189211	189734	190258

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
78250	55	6	11	17	22	28	33	39	44	50
79680	54	6	11	16	22	27	32	38	43	49

N.	D	1	2	3	4	5	6	7	8	9
81170	53	5	11	16	21	27	32	37	42	48
82120	52	5	10	16	21	26	31	36	42	47

N.	0	1	2	3	4	5	6	7	8	9
830	9190781	191304	191827	192350	192873	193396	193919	194442	194965	195488
831	196010	196533	197055	197578	198100	198623	199145	199667	200189	200711
832	201233	201755	202277	202799	203321	203842	204364	204886	205407	205929
833	206450	206971	207493	208014	208535	209056	209577	210098	210619	211140
834	211661	212181	212702	213222	213743	214263	214784	215304	215824	216345
835	216865	217385	217905	218425	218945	219465	219985	220504	221024	221543
836	222063	222582	223102	223621	224140	224659	225178	225698	226217	226736
837	227255	227773	228292	228811	229330	229848	230367	230885	231404	231922
838	232440	232958	233477	233995	234513	235031	235549	236066	236584	237102
839	237620	238137	238655	239172	239690	240207	240724	241242	241759	242276
840	242793	243310	243827	244344	244860	245377	245894	246410	246927	247444
841	247960	248476	248993	249509	250025	250541	251057	251573	252089	252605
842	253121	253637	254152	254668	255184	255699	256215	256730	257245	257761
843	258276	258791	259306	259821	260336	260851	261366	261880	262395	262910
844	263424	263939	264453	264968	265482	265997	266511	267025	267539	268053
845	268567	269081	269595	270109	270622	271136	271650	272163	272677	273190
846	273704	274217	274730	275243	275757	276270	276783	277296	277808	278321
847	278834	279347	279859	280372	280885	281397	281909	282422	282934	283446
848	283959	284471	284983	285495	286007	286518	287030	287542	288054	288565
849	289077	289588	290100	290611	291123	291634	292145	292656	293167	293678
850	294189	294700	295211	295722	296233	296743	297254	297764	298275	298785
851	299296	299806	300316	300826	301336	301847	302357	302866	303376	303886
852	304396	304906	305415	305925	306434	306944	307453	307963	308472	308981
853	309490	309999	310508	311017	311526	312035	312544	313053	313562	314070
854	314579	315087	315596	316104	316612	317121	317629	318137	318645	319153
855	319661	320169	320677	321185	321692	322200	322708	323215	323723	324230
856	324738	325245	325752	326259	326767	327274	327781	328288	328795	329301
857	329808	330315	330822	331328	331835	332341	332848	333354	333860	334367
858	331873	335379	335885	336391	336897	337403	337909	338415	338920	339426
859	339932	340437	340943	341448	341953	342459	342964	343469	343974	344479
860	341985	345489	345994	346499	347004	347509	348013	348518	349023	349527
861	350032	350536	351040	351544	352049	352553	353057	353561	354065	354569
862	355073	355576	356080	356584	357087	357591	358095	358598	359101	359605
863	360108	360611	361114	361617	362120	362623	363126	363629	364132	364635
864	365137	365640	366143	366645	367148	367650	368152	368655	369157	369659
865	370161	370663	371165	371667	372169	372671	373172	373674	374176	374677
866	375179	375680	376182	376683	377184	377686	378187	378688	379189	379690
867	380191	380692	381193	381693	382194	382695	383195	383696	384196	384697
868	385197	385698	386198	386698	387198	387698	388198	388698	389198	389698
869	390198	390697	391197	391697	392196	392696	393195	393695	394194	394693
870	395193	395692	396191	396690	397189	397688	398187	398685	399184	399683
871	400182	400680	401179	401677	402176	402674	403172	403670	404169	404667
872	405165	405663	406161	406659	407157	407654	408152	408650	409147	409643
873	410142	410640	411137	411635	412132	412629	413126	413623	414120	414617
874	415114	415611	416108	416605	417101	417598	418095	418591	419088	419584
875	420081	420577	421073	421569	422065	422562	423058	423553	424049	424545
876	425041	425537	426032	426528	427024	427519	428015	428510	429005	429501
877	429996	430491	430986	431481	431976	432471	432966	433461	433956	434450
878	434945	435440	435934	436429	436923	437418	437912	438406	438900	439395
879	439889	440383	440877	441371	441865	442358	442852	443346	443840	444333

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
42720	52	5	10	16	21	26	31	36	42	47
44320	51	5	10	15	20	26	31	36	41	46

N.	D	1	2	3	4	5	6	7	8	9
85990	50	5	10	15	20	25	30	35	40	45
87730	49	5	10	15	20	25	29	34	39	44

N.	0	1	2	3	4	5	6	7	8	9
880	444827	445320	445814	446307	446800	447294	447787	448280	448773	449266
881	449759	450252	450745	451238	451730	452223	452716	453208	453701	454193
882	454686	455178	455671	456163	456655	457147	457639	458131	458623	459115
883	459607	460099	460591	461082	461574	462066	462557	463049	463540	464031
884	464523	465014	465505	465996	466487	466978	467469	467960	468451	468942
885	469433	469923	470414	470905	471395	471886	472376	472866	473357	473847
886	474337	474827	475317	475807	476297	476787	477277	477767	478257	478747
887	479236	479726	480215	480705	481194	481684	482173	482662	483151	483641
888	484130	484619	485108	485597	486085	486574	487063	487552	488040	488529
889	489018	489506	489995	490483	490971	491460	491948	492436	492924	493412
890	493900	494388	494876	495364	495852	496339	496827	497315	497802	498290
891	498777	499264	499752	500239	500726	501213	501701	502188	502675	503162
892	503649	504135	504622	505109	505596	506082	506569	507055	507542	508028
893	508515	509001	509487	509973	510459	510946	511432	511918	512404	512889
894	513375	513861	514347	514832	515318	515803	516289	516774	517260	517745
895	518230	518716	519201	519686	520171	520656	521141	521626	522111	522595
896	523080	523565	524049	524534	525018	525503	525987	526472	526956	527440
897	527924	528409	528893	529377	529861	530345	530828	531312	531796	532280
898	532763	533247	533731	534214	534697	535181	535664	536147	536631	537114
899	537597	538080	538563	539046	539529	540012	540494	540977	541460	541943
900	542425	542908	543390	543873	544355	544837	545319	545802	546284	546766
901	547248	547730	548212	548694	549176	549657	550139	550621	551102	551584
902	552065	552547	553028	553510	553991	554472	554953	555434	555916	556397
903	556878	557358	557839	558320	558801	559282	559762	560243	560723	561201
904	561684	562165	562645	563125	563606	564086	564566	565046	565526	566006
905	566486	566966	567445	567925	568405	568885	569364	569844	570323	570803
906	571282	571761	572241	572720	573199	573678	574157	574636	575115	575594
907	576073	576552	577030	577509	577988	578466	578945	579423	579902	580380
908	580858	581337	581815	582293	582771	583249	583727	584205	584683	585161
909	585639	586117	586594	587072	587549	588027	588505	588982	589459	589937
910	590414	590891	591368	591845	592322	592800	593276	593753	594230	591707
911	595184	595660	596137	596614	597090	597567	598043	598520	598996	599172
912	599948	600425	600901	601377	601853	602329	602805	603281	603756	6012.32
913	604708	605183	605659	606135	606610	607086	607561	608036	608512	608987
914	609462	609937	610112	610887	611362	611837	612312	612787	613262	613736
915	614211	614686	615160	615635	616109	616583	617058	617532	618006	618181
916	618955	619429	619903	620377	620851	621325	621799	622272	622716	623220
917	623693	624167	624640	625114	625587	626061	626534	627007	627481	627954
918	628427	628900	629373	629846	630319	630792	631261	631737	632210	632683
919	633155	633628	634100	634573	635045	635517	635990	636462	636931	637406
920	637878	638350	638822	639294	639766	640238	640710	641181	641653	642125
921	642596	643068	643539	644011	644482	641953	645425	645896	646367	646838
922	647309	647780	648251	648722	649193	649664	650135	650605	651076	651546
923	652017	652488	652958	653428	653899	654369	654839	655309	655780	656250
924	656720	657190	657660	658130	658599	659069	659539	660009	660478	660948
925	661417	661887	662356	662826	663295	663764	664233	664703	665172	665641
926	666110	666579	667048	667517	667985	668454	668923	669392	669860	670329
927	670797	671266	671734	672203	672671	673139	673607	674076	67451	675012
928	675480	675948	676416	676884	677351	677819	678287	678754	67922.	679690
929	680157	680625	681092	681559	682027	682494	682961	683428	68389	684362

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
89550	48	5	10	14	19	24	29	34	38	43

N.	D	1	2	3	4	5	6	7	8	9
91130	47	5	9	14	19	24	28	33	38	42

N.	0	1	2	3	4	5	6	7	8	9
930	9684829	685296	685763	686230	686697	687164	687631	688097	688564	689030
931	689497	689963	690430	690896	691362	691829	692295	692761	693227	693693
932	694159	694625	695091	695557	696023	696488	696954	697420	697885	698351
933	698816	699282	699747	700213	700678	701143	701608	702074	702539	703004
934	703469	703934	704399	704863	705328	705793	706258	706722	707187	707652
935	708116	708581	709045	709509	709974	710438	710902	711366	711830	712294
936	712758	713222	713686	714150	714614	715078	715542	716005	716469	716932
937	717396	717859	718323	718786	719249	719713	720176	720639	721102	721565
938	722028	722491	722954	723417	723880	724343	724806	725268	725731	726193
939	726656	727118	727581	728043	728506	728968	729430	729892	730354	730816
940	731279	731741	732202	732664	733126	733588	734050	734511	734973	735435
941	735896	736358	736819	737281	737742	738203	738664	739126	739587	740048
942	740509	740970	741431	741892	742353	742814	743274	743735	744196	744656
943	745117	745577	746038	746498	746959	747419	747879	748340	748800	749260
944	749720	750180	750640	751100	751560	752020	752479	752939	753399	753858
945	754318	754778	755237	755697	756156	756615	757075	757531	757993	758452
946	758911	759370	759829	760288	760747	761206	761665	762124	762582	763041
947	763500	763958	764417	764875	765334	765792	766251	766709	767167	767625
948	768083	768541	769000	769458	769915	770373	770831	771289	771747	772201
949	77266	773120	773577	774035	774492	774950	775407	775864	776322	776779
950	777236	777693	778150	778607	779064	779521	779978	780135	780892	781348
951	781805	782262	782718	783175	783631	784088	784544	785001	785457	785913
952	786369	786826	787282	787738	788194	788650	789106	789562	790017	790473
953	790929	791385	791840	792296	792751	793207	793662	794118	794573	795028
954	795484	795939	796394	796849	797304	797759	798214	798669	799124	799579
955	800034	800488	800943	801398	801852	802307	802761	803216	803670	804125
956	804579	805033	805487	805942	806396	806850	807304	807758	808212	808666
957	809119	809573	810027	810481	810934	811388	811841	812295	812748	813202
958	813655	814108	814562	815015	815468	815921	816374	816827	817280	817733
959	818186	818639	819092	819544	819997	820450	820902	821355	821807	822260
960	822712	823164	823617	824069	824522	824974	825426	825878	826330	826782
961	827234	827686	828138	828589	829041	829493	829945	830396	830848	831299
962	831751	832202	832654	833105	833556	834007	834459	834910	835361	835812
963	836263	836714	837165	837616	838066	838517	838968	839419	839869	840320
964	840770	841221	841671	842122	842572	843022	843473	843923	844373	844823
965	845273	845723	846173	846623	847073	847523	847973	848422	848872	849322
966	849771	850221	850670	851120	851569	852019	852468	852917	853366	853816
967	854265	854714	855163	855612	856061	856510	856959	857407	857856	858305
968	858753	859202	859651	860099	860548	860996	861445	861893	862341	862790
969	863238	863686	864134	864582	865030	865478	865926	866374	866822	867270
970	867717	868165	868613	869060	869508	869955	870403	870850	871298	871745
971	872192	872640	873087	873534	873981	874428	874875	875322	875769	876216
972	876663	877109	877556	878003	878450	878896	879343	879789	880236	880682
973	881128	881574	882021	882467	882913	883360	883806	884252	884698	885144
974	885590	886036	886481	886927	887373	887818	888264	888710	889155	889601
975	890046	890491	890937	891382	891828	892273	892718	893163	893609	894054
976	894498	894943	895388	895833	896278	896723	897167	897612	898057	898501
977	898946	899390	899835	900279	900723	901168	901612	902056	902500	902944
978	903389	903833	904277	904721	905164	905608	906052	906496	906940	907383

PROPORTIONAL PARTS

N.	D	1	2	3	4	5	6	7	8	9
933	904	5	9	14	18	23	28	32	37	41

N	D	1	2	3	4	5	6	7	8	9
954	504	5	9	14	18	23	27	32	36	41

N.	0	1	2	3	4	5	6	7	8	9
979	9907827	908271	908714	909158	909601	910044	910488	910931	911374	911818
980	912261	912704	913147	913590	914033	914476	914919	915362	915805	916247
981	916690	917133	917575	918018	918461	918903	919345	919788	920230	920673
982	921115	921557	921999	922441	922884	923326	923768	924210	924651	925093
983	925535	925977	926418	926860	927302	927744	928185	928627	929068	929510
984	929951	930392	930834	931275	931716	932157	932598	933039	933480	933921
985	934362	934803	935244	935685	936126	936566	937007	937448	937888	938329
986	938769	939210	939650	940090	940531	940971	941411	941851	942291	942731
987	943172	943612	944051	944491	944931	945371	945811	946251	946690	947130
988	947569	948008	948448	948888	949327	949767	950206	950645	951085	951524
989	951963	952402	952841	953280	953719	954158	954597	955036	955474	955913
990	956352	956791	957229	957668	958106	958545	958983	959422	959860	960298
991	960737	961175	961613	962051	962189	962927	963365	963803	964241	964679
992	965117	965554	965992	966430	966868	967305	967743	968180	968618	969055
993	969492	969930	970367	970804	971242	971679	972116	972553	972990	973427
994	973864	974301	974738	975174	975611	976018	976485	976921	977358	977794
995	978231	978667	979104	979540	979976	980413	980849	981285	981721	982157
996	982593	983029	983465	983901	984337	984773	985209	985645	986080	986516
997	986952	987387	987823	988258	988694	989129	989564	990000	990435	990870
998	991305	991741	992176	992611	993046	993481	993916	994350	994785	995220
999	995655	996090	996524	996959	997393	997828	998262	998697	999131	999566

PROPORTIONAL PARTS.

N.	D	1	2	3	4	5	6	7	8	9
97600	44	4	9	13	18	22	26	31	35	40

N.	D	1	2	3	4	5	6	7	8	9
99850	43	4	9	13	17	22	26	30	34	39

LANDSURVEYING.

1. THE chief purposes of Measuring Land is to find the Superficial Content, or Area, of any given extent, as comprising a portion of the Surface of the Earth, and to delineate a plan of its figure upon the same plane *.

2. THE means usually employed for taking the dimensions of Land, is either by using the measuring chain only, or by using the chain with an angular instrument. The first of these methods of which we are immediately to treat, is the application of Practical Geometry, and the second is that of Trigonometry.

OF MEASURING A STRAIGHT LINE.

3. To find the true distance between two given points, upon the surface of the earth is yet a desideratum in the art of measuring, as the best measurements of this kind which have been made, are allowed only to be approximations to the truth; but as the difference

* THE plane of the horizon is here to be understood; as this, and the Spherical Surface have almost no sensible difference upon the extent of the largest Estate.

in some cases have been so very small, that in regard to all practical uses, and even for the establishment of theory, these justly may be considered perfect. As an instance of this, the base line used in the survey of the meridional arc of England begun in 1784, was at that time measured with glass rods, and after the corrections for variation of temperature, &c. was found to be 27404.01 feet in length; and in 1791, the same line was again measured with a Steel Chain made by Ramsden, was found after making the same corrections, to be 27404.32 feet; the difference of these measurements being only 2.82 inches in 5.228 English miles. However, in the practice of Landsurveying, such accuracy seems never to have been made in any case whatever, whether the obstacles to this may have arisen from the want either of science in the practitioners, or the use of proper instruments.

4. As the chain is the instrument used for directly measuring distances between given objects; it is always necessary to compare it before using with a true standard of feet, a measure which should be provided from a respectable instrument maker. A good method of ascertaining the length of the chain, or rather the difference it may have from the statute length, which we shall here denominate the *chain error*, is first with the standard to mark out the statute length upon the base of some building, pavement, or other immoveable place whereupon the chain can be fairly stretched; so that if there be any difference, this will appear plus or minus of the Statute length. After this verification of the chain, it may then be carried to any other place, but before commencing the intended measurements, mark there also its length upon another fixed place, to which during

the survey it can be at any time applied, and by this means any variations from using afterwards, will be readily known.

5. The chain commonly used in taking the dimensions of land, consists of one hundred links, each of which is joined to the next by three rings that are always included in the length of every link, and the whole is made of iron wire, excepting the brass counters hung at every tenth. Besides the chain, a rod called an offset-staff, equal in length to ten links, and divided accordingly, is used for measuring short distances. The length of the English statute chain is 66 feet, and the Scots chain is $74\frac{4}{10}$ feet, so that each link of the English chain contains 7.92 inches, and the same of the Scots chain 8.88 inches.

6. The distance which is to be measured should be marked out with picquet staffs or poles, at least one at each extremity, and such extreme points are called *stations*. The operation of measuring any line whatever, requires at least two persons employed, the one to lead the chain, and the other to direct it. The first or leader, is provided with ten pins or arrows, and lays his end of the chain by the direction of the follower in a straight line with the distant station, while the latter holds the other end precisely at the other station. At the end of every chain laid between the stations, the leader sticks an arrow into the ground, at which the follower finds always a precise point to hold his end for the next length, and they proceed in this manner till the whole line is measured, while the follower successively gathers as many arrows as there have been chains laid out upon the distance.

7. All surveys made with the chain only, supposes the ground nearly smooth and level, and if otherwise the conclusions derived therefrom

cannot be correct; for any line having an inclination to the plane of the horizon must be longer than if measured upon it, as such becomes the hypothenuse of a right angled triangle, the difference of level of its extremities the perpendicular, and the horizontal distance the base; therefore, the finding of distances, measured upon sloping and undulated grounds, belongs to the use of angular or levelling instruments, and requires the application of Trigonometry.

OF PRACTICAL GEOMETRY

UPON THE GROUND WITH THE CHAIN ONLY.

8. *To find a point d without a given line* A B, *so that the line* A *d shall be perpendicular to* A B.

SET a staff at any convenient point C, and about half the distance which *d* is required from A B: then with the length C A set another at *b*, and make C *d* in the same line and equal to C *b*, and *d* is the point required; that is, A *d* will be perpendicular to A B.

9. *To make a line* C D *parallel at any given distance to another line* A B.

AT *c* and *d*, any two points taken at pleasure upon A B, make *c b* and *d e* both perpendicular to A B (8.): then mark off *c* C and *d* D, each equal to the given distance, and the line C D will be parallel to the line A B.

10. *To find the distances between two stations* A *and* B, *without measuring between them.*

MAKE C D, at any convenient distance, parallel to A B (9.), and A C and B D both perpendicular to A B (8.); then measure from C to D, between which is the same distance as from A to B.—This

case is frequently used in practice, as the line A B may represent a hedge or other fence whereupon it may be impracticable to measure directly, wherefore the line C D is made on ground free of obstacles equal to A B on which its length can be easily taken.

11. *To continue the line* A B *to any given distance* B E.

At any convenient short distance make C D parallel to A B (9.): set a staff F in the same line with C D, and at the given distance B E: then make F E perpendicular to F D C, and equal to C A or D B, and the point E will be upon the line of A B produced. —This example is applicable to all fences or fronts of buildings, the uniformity of which is broken with trees, projections, or other impediments, and that intercept the eye from directly forming a continued straight line.

12. *To find a point* C *which shall be on a straight line between* A *and* B, *by standing between these objects.*

Let a person with a rod C D lay it from any point C, and first point it to B; next, keeping the end D fixed, let him turn the other towards A; again, keep C fixed, and bring D pointing to B, and so on, pointing the rod alternately upon A and B, until it point upon both these objects at the same time, and C will lie in the line A B.

Otherwise (better by two persons, C *and* D*).*

Let C and D each hold a staff any where at pleasure, between the objects A and B, and first let C put D in a straight line with B; next D, keeping stationary, moves C into a straight line with A: again, C stationary, directs D upon B, and so on alternately, till they both settle upon the straight line A B.—The use of this example is always necessary in the case of rising ground lying between A and B, which

may cover each from the view of the other, as either of these operations may be performed upon a place from which both are seen.

13. *To let fall a perpendicular upon the right line* A B, *from an inaccessible object* D.

Upon the plane mark out the line A B, and make F B perpendicular to A D, and A H perpendicular to B D: then through C the intersection of A H and B F, continue the line D C, by fixing a staff in E on A B, and E D is perpendicular to A B.

Otherwise.

Make A a and B b perpendicular to A B, of any convenient length, but equal to each other, and on A b measure the lengths of $a\,c$ and $d\,b$; then, by proportion, $\overline{a\,c+d\,b} : a\,c :: A\,B : A\,E$

$$\text{or, } \frac{a\,c \times A\,B}{\overline{a\,c+d\,b}} = A\,E$$

$$\text{Also, } \overline{a\,c+d\,b} : d\,b :: A\,B : E\,B$$

$$\text{or, } \frac{d\,b \times A\,B}{\overline{a\,c+d\,b}} = E\,B$$

Hence, by measuring out the length of A E or E B, either will determine the point E.

14. *To find the distance of an inaccessible object* D, *from any given point* A.

Through A form the line A B, upon which let fall the perpendicular D E (14.), and measure on D E any convenient distance E G; next make C G F parallel to A B, and measure the length of C F: then, by proportion (Theor. 16.), $\frac{1}{2}\overline{A\,B - C\,F} : G\,E :: \frac{1}{2}\overline{A\,E+E\,B} : D\,E$

$$\text{or } \frac{G\,E \times A\,E+E\,B}{\overline{A\,B-C\,F}} = \frac{D\,E.}{2}$$

Otherwise.

FROM the point A mark off A B of any convenient length perpendicular to A D; also B C of any length perpendicular to A B, and find a point E in the line of C D upon A B; then by measuring the respective lengths of A E, E B, and B C, we have by proportion, B E : B C :: A E : A D; or $\frac{BC \times AE}{BE} = AD$.

15. *To find the distance between two inaccessible objects,* A *and* B.

FORM upon the plane the line C D, as nearly parallel as it can be made to A B, and let fall the perpendicular A *a* and B *b* upon the line C D (14.); also find the lengths of A *a* and B *b* (15.), and measure the distance *a b*; then we have $AB^2 = \overline{Bb - Aa}^2 + \overline{ab}^2$ (Theor. 10.), or $AB = \sqrt{\overline{Bb - Aa}^2 + \overline{ab}^2}$.

16. *Through any given point* C *to form a line parallel to another given inaccessible line* A B.

FIRST mark a point D at pleasure, and find another point E in the line A D, which shall be on the continued line of C B; next make E G parallel to B D, and G C F parallel A D, meeting B D in F: set a staff at H, on the mutual intersection of the lines E G and A F: then the line formed through H and C, as I H C K, will be parallel to A B.

OF MEASURING LANDS WITH THE CHAIN ONLY,

AND PLANNING THEREFROM.

17. IN lands that are bounded by straight lines which are formed by hedgerows, trees, and drains, the practise is under such circumstances to measure the respective lengths of the sides by some of the preceding methods (10, &c.); but where these are free of obstacles, each part may be measured directly upon itself.

18. THE Planning or Protracting of Figures from actual measurement requires the use of several drawing instruments, as compasses, drawing pen, parallel rulers and scales, each of which we shall describe severally, without supposing the use which may have been already made of these in the Practical Geometry.

The *compasses*, (Fig. A.) are usually made of silver or brass, having the joints and points of steel; the joint at the top is a steel axle, upon which both the legs turn, and is provided with a turn-screw for lessening or increasing the friction of this part. The motion of the legs upon the axle should be uniform and smooth, so as to keep steadily any position given to them, without springing or starting back, and the whole well polished, which makes them be easily kept clean. The point of one of the legs is transmoveable, in the place of which can be substituted, singly, two other parts, called pencil and ink legs, the first of which is used occasionally for describing or drawing circles and arcs in lead, and the other the same in ink. When the compasses have both the legs fixed they are called *dividers*.

The *hair compasses*, Fig. B, have a movement at the screw *a* in the middle of one of the legs, by which the point of it after being put near to any distance, can be brought either backward or forward exactly upon the line of division.

The *drawing pen*, Fig. C, is for drawing lines in ink. The ink is put between two bent steel blades, which by the screw *b* can be made nearly to meet at the points; so that the ink is let out less or more during the time of drawing, according to the required strength of the line. The ink point of the compasses is constructed in the same manner. The head of the drawing pen is usually made to screw out with a steel point affixed to it, which is used for pricking or tracing off the first protraction upon clean paper.

The *parallel ruler* is for drawing mechanically parallel lines at any given distance from each other, by one of the blades moving parallel to the other while this is kept fixed. The best movement of this kind is the moving blade to go out perpendicular to the resting one, as Fig. D; but this instrument is as often constructed to have its movement obliquely, as Fig. E. A surveyor should be provided with three of these, the longest two feet in length, the next one foot, and the other six inches, and the two first are better if made of brass, and the last of ivory. As this instrument is of constant use to the draftsman and surveyor, it is of the utmost importance to verify it, which may be done in the following manner: first open the blades at any convenient distance, and lay the instrument in this position upon the paper, and along each blade draw a straight line, as A B and D C; next reverse the instrument by laying the edge by which A B was drawn, upon D C, and bring the other upon A or B, for instance upon B; then, keeping the first steady upon D C, draw a line through B, which, if the ruler is just, will coincide with A B, but if otherwise, will be B *a*

or B a', and the angle a' B A or A B a will be double the angle of the error which the blades make with each other: for let D b be parallel to A B, and the angle C D b be the error of the instrument from parallelism; now if it be reversed, and the edge put upon D b instead as above upon D C, the other will describe B b', making the angle A B b', equal to the angle C D b, for D B and A B are parallel: but again, as first, make the edge coincide with D C instead of D b, and draw by the other edge the line B a, then b' B a must be equal also to C D b, for D C and b' B are parallel, and consequently equal to A B b': hence the angle A B a is equal to twice the angle C B b.

The T *square*, used by architects, makes very good parallels, if made to slide along the smooth edge of a drawing board, or a straight-edge laid and kept steady upon the paper. Sometimes these squares are provided with a revolving head besides the square one, which can be fixed stationary by a screw through it, to any given angle with the blade, by which, lines oblique to the side of the board may be drawn through any given point.

The *scales* are usually made of brass or ivory, but better if of brass, as this metal bears the pressure and point of the compasses without injuring the divisions so much as those of ivory, especially the diagonal scales, which are used only by applying the compasses upon them. This instrument, Fig. E, contains so many primary or large divisions, as 1, 2, 3, 4, 5, &c. each of which may represent the distance of one mile, one chain, one hundred feet, or any other measure fixed upon previous to using. One or both of the extreme divisions of these is divided into ten equal parts, by diagonal lines drawn parallel to each other, but crossing obliquely eleven other parallel lines, which forms the breadth of the scale, and also divides it into ten equal parts. It will appear, that by this mode of division and position of the parallel

lines, the 100th part of one of the primary segments may be obtained; for first beginning at 0, and looking up the perpendicular line of division the intercepted parts between it and the first diagonal 0 1 are gradually lengthened, till the last or uppermost which is equal to one tenth of the primary segment; so that by this construction, the perpendicular is equal to 10, the first subdivision is equal 1, the first division upon the perpendicular is equal 1, and call the least intercepted part between the perpendicular and first diagonal a, we then have by proportion (Theor 16.), $10 : 1 :: 1 : a$, or $a = \frac{1}{10}$ of the subdivision; and every subdivision being the tenth part of the primary segment, consequently a is equal 100th part of the same segment.

In planning grounds which have been measured with an hundred linked chain, the above division of the primary segment is used, as each of these represents 100 links, or one chain length, and the diagonal shows all the intermediate numbers of links from 1 to 100. As an example, let the length of 232 be required from the scale; then we have first two primary segments equal to 200, and three subdivisions equal to 30, now if we add the second intercepted part from the bottom, we have collectively three parts equal to 232; but which in whole can be found on the second line from the bottom, being intercepted between the third diagonal and perpendicular 2. The same may be shown of any other number; for the digit part of the number is always intercepted between a diagonal and the perpendicular, upon the parallel line of the same name from the bottom.

Besides the above application of the diagonal division of the scale in representing the parts of 100 links, it is equally applicable to any other measure whatever: for instance, let the above primary segment be subdivided only into eight equal parts instead of ten, then by the same proportion we have the eightieth part of the same division, so

that this scale will answer to miles, furlongs, and chains. The same may be shown in feet, inches, and tenth parts of an inch; for if the primary division is divided into 12 parts, and the height of the diagonal as before into 10, we have the first representing feet, the second inches, and the third tenth parts of an inch.

19. As the form of a crooked line cannot be derived from measuring lineally the required part only, excepting this be a portion of a known curve or circle, it hence becomes always necessary to refer the measure of such lines to a common standard, upon which the forms of these will appear. The most convenient measure for this purpose, is a straight line made near and to lie opposite the crooked line; for let it be required to find the form of the line A *b c d* E: with two or more poles mark out the straight line F K, opposite and conveniently near to A *b c d* E, and measure alternately the greatest and least distances between them, as the perpendiculars A F, *b g*, *c h*, *d i*, E K; also measure the parts F *g*, *g h*, *h i*, *i* K, which together will be the necessary dimensions for constructing the crooked line A *b c d e* E; as draw first the line F K, and mark out the distances respectively of the points F, *g*, *h*, *i*, K; next set off the perpendiculars A F, *b g*, *c h*, *d i*, and E K, according to the measurement of each, and join their extremities A, *b*, *c*, *d*, E, which will be the form of the line required. In curved lines, it is obvious, the nearer the perpendiculars are to each other, the form of the curve will be the more correctly described.

The line F K is called a *station line*, and all the perpendiculars measured to the boundary, from it, are termed *offsets*.

20. When crooked or curved lines recede so far from the station line E L, that the offsets are inconveniently lengthened for measure-

ment, as of the part C D opposite to G L, it is better, in this case, to mark out another line G K, joining and making an angle with E G, but nearer to G D than G L, by which the offsets on this part will be much shorter than if measured upon the continuation of E G.

21. The relative position of two lines E G, G K, forming any angle with each other, is found by measuring the subtense * of the angle E G K or K G L; for mark off the equal distances G *h*, G *k*, and measure the subtending line *h k* between them, the lines G *k*, G *h*, *h k* are the dimensions of a small iscoles triangle G *h k* upon the required angle.

Protraction. Draw E L, and mark off the distances E G=360 and G *k*=100; upon G *k* as a base, describe with G *h*=100 and *h k*=150 the triangle G *h k*, and produce G *h* as far as K, then E G and G *k* are in true position to each other. When the angle E G K is very obtuse, it is better to take the measure of the subtense *h k* of the angle K G L, as the point *h* can be more accurately determined, because the intersection of the sides G *h* and *h k* are not so oblique as in the other.

In making the actual measurements of land, an instrument called a cross staff is sometimes used, for marking out with greater expedition the offset lines perpendicular to the station line, than could be done by geometrical construction. This instrument is constructed with four sights, fixed at right angles upon the head of a staff of convenient length, which is shod with iron for sticking into the ground. It is applied by being placed upon the chain or station line with two

* The subtense is a line joining the two sides which form the angle at equal distances from the angular point.

of the sights in that direction, and by looking through the other two, a mark on a perpendicular line to the chain is thus found upon the boundary, where the offset is to be measured; or it is moved backward or forward upon the station line till the perpendicular formed by the intersecting sights falls upon any given point: but as this instrument is not provided with any adjustment for making its plane parallel to the horizon, its operations should always be limited to very short distances; therefore practical surveyors prefer as equally accurate, to keep their station lines as close to the boundaries as can be done, which makes the offsets only be a few links in length, and are taken perpendicular, by the eye first laying the offset rod as nearly perpendicular across the chain upon the station line, and then measuring with it forward to the boundary in that direction. By this method, or using the cross staff, the station line is usually measured, and the offsets upon it are taken at the same time.

22. The operation of measuring a line with offsets as E K, is performed first by measuring with the offset staff the offset A F=15 links, next stretch the chain from the station F towards the station K as described in (Art. 6.), and proceed measuring till some part of the chain lie opposite the point *b* upon the boundary to which the next offset is required; then find the point *g* upon the chain by (Art. 21.), so that *b g* will be perpendicular to F K, and measure *b g*=25 links; also note the number 120 the distance in links found by the chain, between F and *g*. In the same manner continue till the chain lie also opposite *c* and *d*, where likewise as above measure the offsets *c h*=17 links, and the distance of *h* from F is found 230 links, and the offset *d i*=30 links, and *i* distant from F is 360 links. Lastly, the whole distance F K is found to be 513 links, and the offset K E at the sta-

tion K is equal 10 links, which measurements together make the dimensions of the boundary A *b c d* K.

As the measurement of the line F K represents a straight line, it is necessary that this be actually measured as such; for if those employed have deviated to the right or left of it, this measurement will not only be lengthened, but likewise all the offsets from it be measured long or short of the true dimension according as the deviation approaches to, or recedes from the boundary; therefore the persons who direct and lead the chain, should be well instructed and practised, till able to keep it at every point in a right line between the picquet staffs, which mark out the course under measurement.

In the case where the ground upon which offsets are required is sloping or rising from the station line to the boundary, it is better to measure these from the highest extremity to the lowest, by first putting the one end of the rod upon the surface, and keeping it as nearly level by the eye as possible, and from the other suspend a plummet or drop a small stone, which will mark the horizontal distance of the rod upon the sloping surface, and also a point from which to measure the succeeding length, In the same manner continue till the whole length of the offset is measured, which will be sufficiently near to the true horizontal distance.

It is better, as on the figure, to write the distance upon the station line where the offsets are taken, in links, as F g=120, F h=230, F i =300, &c. instead of noting the chains and links separately. In the event of a station line exceeding ten chains in length, it is necessary at the end of the tenth, or 1000 links, that the whole arrows which have been now taken up by the follower be again returned to the leader, who sticks the first at the end of the eleventh chain, and both proceed as before; but in the act of returning the arrows, the follower

marks the place of the tenth arrow till the eleventh is stretched, and the leader sticks in the eleventh arrow; or otherwise if the eleventh chain is stretched before changing, the leader marks the place of the end of the eleventh till the follower brings up the arrows, who sticks one at this point, and gives the remaining nine to the leader. However, instead of marking the places of the tenth arrow or the end of the eleventh chain, the writer of this uses an eleventh arrow, but which has a peculiar mark from the other, as half the length, &c. which the leader always reserves to be put in at the end of the eleventh chain length, so that when the leader at this point receives the arrows, he lifts it again and sticks one of the ten in its place, and again carries it in reserve for the same purpose at the next change, by which means are avoided the accidents of losing the point, which may happen from not marking the place of the arrow sufficiently, and afterwards while changing not keeping the spot in view. It is obvious, at the end of whatever number of chains measured by this method, that this eleventh arrow is not taken into the enumeration; but has been held merely as a mark during the changes made on the whole line.

In measuring with the chain, the assistants should be very careful to give out its full length, by keeping the handle within two or three inches of the point of the arrow, and holding this perpendicular while sticking it into the ground, which is easily done by the leader putting all the fingers of the right hand through the handle, and the arrow on the outside between the handle and thumb. On the other hand, the follower should hold the chain precisely at the arrow of his end, until he finds the leader has put another in a direct line with the stations.

23. If the sides of a field which is to be measured is found to assume

the figure of a triangle, as A B C, first plant a picquet staff at each of the points *a*, *b*, *c* as stations, and at such distances from the fences A B, B C, C A, as the lines *a b*, *b c*, *c a* can be measured without interruption either from the inequalities in the surface, or drains, trees, &c. which may be upon them. If the fences are straight lines and beginning at *a*, measure the offsets *a a'*=10 links, and the station line *a b*=420 links, also the offset *b b'*=20 links: next take the offset *b b''*=17 links and measure the straight line *b c*=345 links, also the offset *c c'*=25 links: then measure the offset *c c''*=12 links, and the station line 475 links, and the offset *a a''*=14 links.

But if the boundary of the field consist of crooked fences, and forming a figure nearest also to the triangle, as E F G, plant the stations *e*, *f*, *g*, under the same circumstances as shown above for *a*, *b*, *c*, and measure the station lines *e f*, *f g*, and *g e*, with the necessary offsets upon each as shown in the figure, and exemplified in (Art. 22.) which is at every turning and recess of the boundary from the station lines.

Protraction of the triangular field having straight sides. First construct the triangle *a b c* (Prob. 1.), for example as in the figure from a diagonal scale of one half inch to the chain or 100 links, making the side *a b*=420, *b c*=345, and *c a*=475; next set off perpendicular to *a b* the offsets *a a'*=10 and *b b'*=20, and draw indefinitely the line *a' b'*. In the same manner mark out the offsets *b b''*=17, *c c'*=25, *c c''*=12, and *a a''*=14, drawing the straight lines *b''* *c'* and *c''* *a''* intersecting each other and *a' b'* in the points A, B, C, and the figure A B C is the true boundary of the field measured.

Protraction of the triangular field having crooked sides. As above construct the triangle *e f g*, making the side *e f*=420, *f g*=345, and *g e*=475; next mark off upon the station line *e f* the offset *e e'*=12, and the distances 80, 130, 230, 330, and 400 from the station *e* upon

the line *e f*; then from these points set out perpendicular the offsets which were measured respectively from them, as 15 at 80, 30 at 130, 8 at 280, 26 at 330, 15 at 400, and 15 at the station *f*; in the same manner proceed to mark out the intermediate distances, at which offsets were taken upon the other station lines *f g* and *g e*, and at the same time as above constructing the offsets perpendicular to the respective station lines of each; after which join the extremities of all the offsets, and thereby forming the crooked lines E F, F G, and G E being together the boundary of the field, which will be correctly delineated.

24. As another example of surveying a crooked sided field, for it is such that oftenest occur in practice, let the field A B C D, by placing the stations *a*, *b*, *c*, *d*, be resolved into a polygon of four unequal sides, as the most convenient figure for taking the dimensions upon its boundary. The surveyor in this case may either place all the stations before measuring, or place and measure them in succession. After all the station lines and the necessary offsets upon each are measured, it is requisite that both the diagonals *a c* and *b d* be also taken, by which the polygon is divided into two triangles, being either *a b c* and *a c d*, or *a b d* and *b d c*.

Protraction. First draw *a c* or *b d*, but in this example let *a c* be drawn, and upon it as a base construct the triangle *a b c* and *a c d*, according to the measured lengths of the respective sides of each, as marked upon the figure. As in (Art. 23.) set off the offsets upon the sides of the polygon *a b c d*, and the extremities of these will mark as before the boundaries of the field.

In this example a good proof of the accuracy of the measurements of the station lines are afforded from the measurement of the diagonal *b d*; for if the length of *b d* be measured by the scale from which the

other parts of the figure are constructed, this should exactly be agreed with its measured distance, and which is an instance that shows the necessity of providing similar proofs for the measurement of every figure whatever, whereby the results to be derived from them may be safely relied upon.

25. FIELDS which have more than four sides commonly require as many station lines as each has sides, as A B, B C, C D, D E, and E A, which are measured after the same manner as above described in the fields of three and four sides, but this field will require two diagonals to be measured between the opposite angles, as B D and B E, or C E and A D, for protracting it; for by the figure there are three triangles to be made, as C D B, D B E and A B E, the two first of which can be constructed upon the base B D, but the other requires the other diagonal B E for its base.—It farther may be made evident, that every figure having more than three sides, requires as many diagonals measured between the opposite angle as the figure has sides more than three; for let other two sides be formed, as E F and F A, which will increase the same figure to six sides, as A B, B C, C D, D E, E F, F A, and then a third diagonal A E will be necessary for a base, upon which to construct the triangle A F E.

It is upon the above principles that the most accurate measurements of land are made with the chain only, that is first by circumscribing or inscribing the boundary with station lines, and forming diagonals between the opposite angles by which the figure assumed by the station lines is divided into triangles. However, as the most careful measurements are not perfect, nor each have always the same ratio to its true distance, a choice of position to each other and the proportions of the sides is necessary to be considered,

so as to construct a series of triangles to have the same accuracy with which they may have been measured. As an instance of this practical remark, for it is in practice alone it is admissible and by which it is discovered; let a series of triangles be formed, such as whose sides have great differences with each other, and in this case the intersection of the sides at one of the angles will cut extremely oblique, of which the true point of intersection cannot be determined: but on the contrary if the sides of all the triangles have small differences with each other, the intersection of all the angles will be nearly the most direct possible, and upon each the point of intersection will be definitely marked.—Hence the nearer equilateral are the triangles, and equivalent to each other, the measurements which have been made of them can be the most accurately protracted.

26. The measurements of every polygonal or many sided figure may be verified by diagonals measured to every second opposite angle from each of these alternately; for let the polygonal field A B C D E F G H be constructed from the measurements of the triangles B C D, A B D, A D H, H D F, H E F and H F G, and besides these let the diagonals A C, A E and E G have been also measured between every second opposite angle and from each alternately: it appears that A C is a base to both the triangles A B C and A C D, and A E is a base for the triangles A D E and A H E, and also E G is a base for the triangles E H G and E F G, by which each of the quadrilateral figure B C D A, A D E H, and H E F G are verified by its respective diagonal (Art. 24).

27. Where deep recesses or great bends are in the fences, as at E, to which the distances are too great to measure with the off-

set staff from the station line A B, construct a triangle within the recess upon a base from two known points of the station line, as C D E, upon which from two of its sides as C E and E D, shorter offsets can be taken to the boundary.

If a recess consists of two straight sides as *a* F, F *b*, mark the number intersected by the continuation of them upon the station line, as 326 and 410, also marking other two convenient points upon it, as 340 and 400 and measure the distances 116 and 108 from each to F as on the figure, whereby F is constituted the apex of a small triangle, having the difference of 340 and 400 for its base which is a part of the station line A B.

Instead of a triangle being formed upon the station line for measuring the boundary of a recess, a trapezium or any other polygon may sometimes be more suitable for this purpose, as H I K L, and by measuring the diagonal I L or H K, as in (Art. 24.), is divided into two triangles, being either H I K and H K L, or H I L and L I K. In the same manner as shown in (Art. 25.) the boundary of any recess whatever may be inscribed or circumscribed by station lines forming a polygon of any number of sides, but one of which must be always a known part of the station line A B.

In the case of a pond of water or other obstacle by which it is impracticable to measure the diagonal M P or N O in the polygon inscribing the recess, it is necessary in this example to measure the subtense of the angles L M O, M N P, M O P, and O P N, as in (Art. 21.), by which the angular position of the lines M O, N P, and O P will be known.

Let it here be observed, of this method for finding the angular positions of the sides of a polygon by measuring the subtense at short distances from the angular point, should only be applied to such cases as

the above, in which [illegible] do not exceed two or three chains; for a small [illegible] neglected upon the subtense will make a considerable [illegible] true position at the extremity of a long line. I [illegible] of the [illegible] of many *respectable* measurers of land, who use this method [illegible], and even give it an ignorant preference to the use of angular instruments; but as a better proof than the [illegible] of practitioners, let the angle M N P be taken by measuring the subtense N P, the true length of which is 150 links at the distance of 100 links from the angular point N, and suppose N P and N M equal, and each [illegible] links, then by (Theor. 16.) we have 100 : 150 :: 1000 : M P=1500, the true length of M P: now let an error of one link take place in the measurement of the subtense, that is, instead of being measured the true distance 150 let this be only 149; then we have by the same proportion 100 : 149 :: 1000 : M P=1490, which is 10 links short of the true length of M P: again let the subtense be measured 151 instead of 150, then as above 100 : 151 :: 1000 : M P=1510, which is 10 links too long. Hence in the same proportion will the error be multiplied as the line is longer than at the point where the subtense was measured.

When the angular position of a fence Q R is required upon the boundary, mark its intersection S upon the station line by placing a pole upon the chain in a line with Q R, and also mark another point T; then measure with the offset staff the distances S Q and T Q, which will constitute the triangle S Q T, having its base S T a known portion of the station line and one of its sides S Q in the same line of Q R.

Protraction. First mark out the distance s S=476 and s T=565, and with the sides 35 and 42 construct the triangle as upon the figure: then produce S Q beyond the boundary to R, and Q R is in true angular position to the boundary q r.

Where buildings or other angular [illegible] are [illegible] to the boundary, as a house G, [illegible] upon the station line as at 520 and 606, [illegible] point from the same intersections as 50 and 70. If the [illegible] of the building be required, measure its length 64 and breadth 32, and at the same time noting the meeting of fences or other lines upon it, as represented on the figure by 20 and 30 [illegible] gular point to the lines marked fence.

Protraction. First lay off from the station A the distance 520 and 606, and from these points, with the distances 50 and 70 as sides, construct as in the figure a triangle, the apex of which or angular point opposite the station line is upon the corner of the building to which these distances were measured.—Next produce the sides 50 and 70, and lay off upon the continuation of 50 the length 64, and upon the continuation of 70 the breadth 32; then draw the opposite sides parallel, and the building G is delineated both in position and magnitude.

If the position of a rectangular building W, lying nearly parallel to the station line be required, first mark the points of intersection by producing its opposite sides $u\,u'$ and $v\,v'$ upon the chain as U and V; then measure the distances U u and V v, and its breadth $u\,u'$ or $v\,v'$ and length $u\,v$.

Protraction. From the station a lay off the distance of a U$=$ 343 and a V$=$403; next from the point U with the distance 55 describe an arc $u''\,u\,u'''$, and from the point V describe another arc $v''v\,v'''$, and draw $u\,v$ touching both the arcs: then draw the lines U $u\,u'$ and V $v\,v'$ both perpendicular to $u\,v$, and make $u\,u'$ or $v\,v'$ each equal the breadth 34 and draw $u'\,v'$, which will delineate the building W.—It is to be noticed by the construction of the above, the building is supposed rectangular, by which the length will be found 58 as measured.

When [illegible] station line A B is intercepted by a small piece [illegible] or other impediment as X, over which the line may be continued but cannot be measured, measure as far as *c* and set out *c d* perpendicular to A B, by (Art. 8.) or the cross staff, but of such a length as *d e* may [illegible] be set off perpendicular to *d c*; from *d* measure *d e* any convenient distance, also beyond the impediment X, so that *e f* may be made perpendicular to *d e* and be produced to *f* on the line A B.—Now if *e f*, when *f* is found upon the line of A B, be equal to *d c* and the angle *e f* B be a right angle, the distance across the obstacle X is equal to the measured distance of *d e*. Hence by measuring *f* B the whole length of A B is equal to A C+*d e*+*f b*.

It occasionally is necessary to cross or pass through a fence C D with the station line A B as at *a*, *b*. If this fence be a hedge, in most cases the chain may be passed between the stems of it upon the line A B, having the chain stretched as in open ground; but if it be a wall of stone, turf, &c. through which the chain cannot be passed, first measure to the side at *a* as 278, next with the offset staff laid horizontally across the wall in the line of A B, and with its end adjusted by a plummet exactly over *a*, find the number of links as 6 upon the rod by again suspending the plummet above *b*, which add to 278 making together a distance 284 from A to *b*; then, subtract this from 300 and the remainder is 16, which difference lay off with the chain from *b* towards B, and this will mark the place of the third arrow from A to B, beyond the wall.

28. After having described the methods of measuring the station lines with offsets upon them, the joining with diagonals the opposite angles formed by the station lines inscribing or circumscribing a single field, and also the various figures of the boundary and objects connected

therewith; it may be farther useful to illustrate these different examples by another more general, showing the practice of measuring and delineation of several fields adjoining each other.

The survey of the five fields within the boundary A B C D E F G, was begun by first fixing the station *a*, so that the extremities A and B of the boundary A B were both seen from it; we next fixed *b* in such a manner as the line *a b* could be measured free of obstacles by crossing the rivulet; after having *b* fixed we went to the meeting of the river and the boundary at E, and placed the station pole at C, by which the line *b c* could be measured without any part of it falling into the river, and also free of interruption upon the banks. The part of the boundary E F being a straight line, it was only necessary to place the station *d* at a convenient distance from it, and to be seen from *b* and *c*, and some part of the line *a b* as at *b'*. Proceeding as far as the corner G, we found that *d* would be seen on a straight line from a point *e*, and into the road at *f*, and also that part of the station line *a b*, which is intercepted in the same inclosure; therefore both *e* and *f* were constituted stations.

The stations *a*, *b*, *c*, *d*, *e*, *f* being now posited, we went along the road till nearly opposite B, where we fixed the station *g*, to which a straight line could be measured from *a*, and likewise another to some part about *h* opposite C. Proceeding till opposite the bend C, the station *h* was placed so that both the straight lines *g h* and *a h* could be easily measured, and also another *h i* towards the river, where *i* was made another station, from which the distances of *h i* and *l i* could likewise be taken.

Having now either circumscribed or inscribed the whole boundary A B C D E F G, by marking out the station lines *a g*, *g h*, *h i*, *i b*, *b c*, *c d*, *d e*, *e f*, and *f a*, and likewise *a b* skirting the rivulet, we con-

sidered [illegible] marked out upon the ground, and found the measurements of these lines as expressed upon the plan, and measured the diagonals [illegible] *h b*, *a e*, *g b*, *b d*, which were necessary for dividing the whole figure of the station lines into triangles: besides these are also measured the proof lines, *c' g*=1012, *c' i*=1046, *c e*=878, and *a' f*=804. While measuring along *a b*, we marked * the point *a'* as the extremity of the line *a a'* which takes the form of the curved fence opposite this line (Art. 19), and also noticed the intersection of this fence with the chain. The intersection of the continuation of the straight fence from the boundary at C [illegible] observed both between *b'* and *c'*, and *d'* and *e'*. The extremity of the straight fence F *h'* was made the apex of the triangle *8' 9' h'*, and likewise the intersection upon *a b* if continued. In measuring the lines *d e*, *e f* and *g h*, the points of the intersecting fences were all observed upon the chain. The angular position of the fence *l' k'*, upon the opposite side of the river to the station line *i b*, was found by marking out the triangle *i' m' n'* from it, having one of its sides *i' m'*, on the same line of *i k*. The methods of taking the measurements of the other parts as expressed upon the figure, have already been sufficiently described in the preceding articles.

Protraction.—The protractions of all surveys are commonly drawn first with a black lead pencil, and afterwards the boundaries and other objects of the lands are drawn with the drawing pen in India ink: but if it is required to preserve the station lines upon the protraction, these gone over with a tracing point, will impress the lead into the paper, so as to leave them afterwards always visible.

* The station points and all others upon the station lines, to which other lines are to be joined, should be marked with small pegs stuck into the hole in the ground made by the pole, by which these points may readily be found afterwards, at least till the survey is completed.

Draw a line of indefinite length, [illegible] from any scale, as in the plate of half an [illegible] every 100 [illegible] upon this as a base, with the sides *a e* and [illegible] construct the triangle *a b e*; next making *a e* another base, construct upon it the triangle *a f e*, upon *e b* the triangle *e b d*, and upon [illegible] the triangle *b c d*. —Again, upon *a b* as a base, construct the triangle *a h b*, also upon *a h* make the triangle *a g h*, and upon *b h* construct the triangle *b i h*, which will conjoin together the series of triangles *a b e*, *a f e*, *b e d*, *b c d*, *a g h*, *a g b*, *b i h*.

From *a* upon *a b*, mark the points *a'*, *b'*, [illegible], *g'*, according to their respective measured distances from *a*, and join *a' e*; now, by applying the length of the proof lines from the scale, *c' g*=1012, *c' i*=1046, *c' e* =878, *a' f*=804, and *a' e*, the degree of coincidence of each will show that of the accuracy with which the survey and the protraction has been made of the station lines, (Art. 24.)

Supposing that each proof line answers to its measured distance upon being applied from the scale; next construct the small or lateral triangles *b' d' e'*, *b' e' c'*, *f' g' h'*, *i' m' n'*, and mark off upon the station lines the respective offsets, and through the extremities of these draw the corresponding boundaries (Art. 19.); also join the points between the station lines where such are intersected by fences, as, join the straight fences *c c'* and F *h'*, which will finish the delineation of the whole of the measurements of these five fields.

In all surveys to be made with the chain only, the lands should be divided by the station lines into the smallest number of triangles, consistent with its general figure: but these should be so disposed to skirt very near the boundaries, by which the windings and other deviations from a right line will be easiest measured by the preceding methods, and also to have the fewest diagonals necessary either for

[redacted] or proof lines to verify it; whereas, were the boundary measured by a great number of station lines, this will multiply both the diagonals and proof lines, and the whole protraction made much more intricate, and certainly less accurate, than by having to unite fewer lines in protraction.

If upon applying the proof lines to a protracted series of triangles as above, and finding any of them not very nearly to coincide with the measured distance, the first thing to be done in this case is to revise the protraction, next the measurements of the proof lines, then the diagonals by which the figures were constructed, and lastly the measurements of the station lines, in one of which the error must necessarily appear.

It is the practice of some surveyors, who pretend to great accuracy, to have a great many proof lines made almost in every position, which by them is termed *tying the survey;* but such labour is unnecessary, and will rather be the cause of inaccuracy than a proof; for after diagonals are measured as single proof lines in the best positions, (Art 25.) all others which are out of this position must be an inferior proof of the accuracy of the survey, because in the practice of measuring every line, it is impossible to form, even with the greatest care, that coincidence which is so evidently demonstrated to take place in theory; therefore an extraordinary number of injudiciously disposed proof lines, can only at the most shew some singular agreement of errors, if taken place upon those of the best position.

29. The best manner of forming a series of triangles for surveying an estate, or other considerable extent of land with the chain only, is first to mark out to the best advantage, that is upon the clearest and most level parts, two or more straight lines quite through the

lands, and nearly parallel to each other, [illegible] A, E F, and [illegible] the distance from each other of three [illegible] the length of the [illegible] which the triangles are required. Then [illegible] these lines, form the triangles A *c b*, A *b a*, *a b f*, B *f* D, *q* D B, [illegible] *d*, C *d b*, *d b e*, *b e c*, *c e* F, and C E D, every one of which will have its base a portion of one of the straight lines, and its apex or opposite angular point in the next; again upon the sides of the triangle, form other, but smaller triangles or polygons, as will be necessary for measuring the boundaries within each primary triangle, as shown in the preceding examples, (Art. 27, 28.)

The advantages of this method arises from the easiness in fixing a straight line almost to any distance, and over every obstacle, and the proof in the protraction which it affords, as the apex of every triangle upon the same side must form another straight line; for it is evident, if either of the sides is measured shorter than the true distance, the apex will fall within the straight line, and if longer, will fall without it, and would require the singular coincidence of an error in both sides, the one shorter, and the other longer than either of their true distances to make this point fall upon the line, excepting in the case of the true distances to both. If it happen that a river, marsh, or other impediment, obstruct the continuation of the measurements of any of the bases or distances between the apex of the triangles, as between *b* and *c*, still the data is sufficient, if the series upon the other lines A B, and E F, have been continued; for the triangles *b d e*, *e c* E, upon E F, and the triangles A *b a*, *a* D B, upon A B, can be constructed upon their respective bases, and the apex of each be made to fall upon the line C D. Here proof lines are not necessary beyond those required for finding the dimensions of the Lands under measurement.

30. When a clump of trees, a lake, or marsh, which lies in the middle either of a single field or farm, through which diagonals as hitherto exemplified, cannot be measured, as A B C D E F, first fix a station pole G, and other stations A, B, C, D, E, F in such a manner as G can be seen and measured from each; then as before measure the lines A B, B C, C D, D E, E F, F A, with the necessary offsets upon them, and also A G, B G, C G, D G, E G, F G, which will finish the dimensions of this field.

Protraction.—Draw any of the bases of the triangle formed by the station lines, as A G, upon which construct the triangle A B G, and conjoin to this the whole series of the triangles B G C, C G D, D G E, E G F, G F A, the last of which, viz. A G F, will be formed after constructing E G F; for after joining A F, the space A G F if the whole is correct, will exactly coincide with the dimensions of this triangle; but if otherwise, some error must have taken place, either in the protraction or measurements.

31. If the obstacles in the middle of the ground be so posited that all the stations which require to be placed around the boundary cannot be seen from one point but from two, as A and B, the same mode of procedure may be adopted as in Art. 30, and exemplified upon this figure, and in the protraction, the same proof of the degree of accuracy will be found by the coincidence of the dimensions of the last protracted triangle.

The same causes for forming one or two points, may form three or even four or five points in the middle of the field, as A B C D E, by which, as in the figure, the whole field can be protracted.

32. When a wood only is the object of measurement, and suppos-

ing the impracticability of measuring [illegible] through it, first place the stations A, B, C, D, E, so as to form the best lines for measuring its boundary, by circumscribing it with station lines; beginning at A measure the dimensions of the small triangle A *i h*, which will measure the angle B A E. In the same manner the triangles B *a b*, C *d k*, *l m* D, and *g f* E, will respectively measure the angles upon which these are formed.

Protraction.—Construct the triangle A *i h*, and produce A *i* to B, and A *h* to E, at B construct upon B *a* as a base the triangle B *a b*, and produce B *b* as far as *d*, and mark off B C: upon C *d* as a base, construct the triangle C *d k*, and produce C *k* to D, and upon D *l* as a base construct the the triangle D *l m*, of which produce *m* D to E: now as a proof of the accuracy of the work, D E and A E should meet each other in E at their measured distances from A and D; and also, besides this coincidence, by marking from E the points *g* and *f*, the side *g f* should likewise exactly correspond with its measured distance, as this triangle measures the angular position of D E and A E.

We have already, in Art. 27, stated the limits by which this method of protracting lines should be restricted.

33. If a road is required to be measured, fix the stations as A, B, C, D, E, at the different turnings, but in such a manner, as usual, that lines can be measured between them, which may either be placed upon the road itself, or upon the outside of it; as in this case, within the road. Beginning from A and measuring A B, the angular position of A B and B C is found by measuring the triangle *a* B *b*; also the angular position of B C and C D is found by marking out the perpendicular *c d* with the cross staff from 400 on C D, and finding

this 11th till meeting [redacted] of B C produced. The same operation is again performed for [redacted] the angular position of C D and D E, by marking with the [redacted] staff from D E the perpendicular *e f*, and measuring *e f* till meeting the line of C D continued, which together with the other necessary measurements and offsets as shown upon the figure, will afford [redacted] sufficient data for planning the road A B C D E. The same method may be applied to any other crooked space of ground whatever, and canals or rivers, as represented upon the plate.

In the foregoing articles, there is given that which I consider as the best practice of measuring land with the chain only, and throughout, such necessary remarks and limitations to the several methods, for ensuring the greatest accuracy of which such operations may be capable; for as already observed (Art. 7.), such suppose the surface to be measured almost perfectly smooth and level, and as this can only be the case partially in whatever country, the deviations from the truth will be proportional as the circumstances depart from or approach to this condition. However, it must be allowed, that for the general purposes of business extreme accuracy is not always required, and therefore what is already taught in surveying may be made occasionally useful;—on the other hand, the use of angular instruments, with the applications of Trigonometry as we shall hereafter treat, will afford the greatest accuracy, be much more expeditious than the above, and applicable to every case, in many of which the use of the chain only is quite insufficient.

OF SUPERFICIAL EXTENSION OR AREAS.

34. The measure of superficial extent or area, is denominated square measure, from a square containing the same area, or being equivalent to any other figured surface; as the square H may contain the same area of the irregular figure I, and if the side of the square H be equal to 1, then its area, and also that of its equivalent I, will each contain a square unit, or be equal to 1^2, which is one square yard, foot, or whatever other name the lineal measure of the side of the square is denominated: also, if the figure I be enlarged in its area, this is always expressed by the number of square units which it may contain.

35. *Of the Square.*—If the side of a square be equal to any number of a given lineal measure, as 2, 3, 4, &c. the area will be 2^2, 3^2, 4^2, &c. that is, equal to the square of the quantity expressing its side; for if each of the sides of the square H′, be divided into the same number of parts which express each of their measures, and the corresponding points of the opposite sides be joined, as in this case into four, then the whole area of the square is formed into 16 squares, each being a square unit of the side, which are equal to 4^2. Hence the rule, *multiply the side into itself, and the product is the area.*

Example.—Required the area of a square whose side is 475.

By Natural Numbers.	By Logarithms.
475	475——2.6766936
475	2
2375	225625=5.3533872
3325	
1900	
225625 the area.	

36. *Of the Parallelogram.*—The same may be shown of the Parallelogram as of a square, for dividing the length and breadth of the parallelogram K, as the side *a* into 5 and *b* into 3, then the whole area is formed into 15 square units, which is equal to 5×3, so that the rule becomes *multiply the length by the breadth, and the product is the area.*

Example.—Required the area of a rectangle, the length of which is 366 and breadth 244.

By Natural Numbers.	By Logarithms.
366	366——2.5634811
244	244——2.3873898
1464	89304=4.9508709
1464	
732	
89304 the area.	

37. *Of the Triangle.*—Every triangle is equal to half of a parallelogram of the same base and altitude (Theor. 9.): therefore in the triangle A B C, the area is equal $\frac{A\,C \times B\,D}{2}$ or, *multiply the base by half of the perpendicular, and the product is the area.*

Example.—Required the area of a tri[illegible]
325 and base 536.

By Natural Numbers.

536
325
2680
1072
1608
2)174200
87100

[illegible]
536
[illegible] —
87100=

Also, if the three sides be denominated by a, b, A, then $\sqrt{\frac{a+b+c}{2} \times \frac{a+b+c}{2} - a \times \frac{a+b+c}{2} - b}$

or, if S equal the semiperimeter (half sum of the
$A^2 = S\ (S-a)\ (S-b)\ (S-c)$ and $A = \sqrt{S\ (S-a)\ (S-}$

This useful formulæ may be easily derived alge
lowing manner *.

$a^2 = A\ D^2 + D\ C^2 + 2\ A\ D \cdot D\ C$ ('
$c^2 = D\ C^2 + D\ B^2$ ('
$b^2 = A\ D^2 + D\ B^2$ ('
and $a^2 + c^2 - b^2 = 2\ D\ C^2 + 2\ A\ D \cdot D$ (

therefore $\frac{a^2+c^2-b^2}{2\ D\ C} = D\ C + A\ D = a$

and $\frac{a^2+c^2-b^2}{2\ a} = D\ C$

but $c^2 - D\ C^2 = B\ D^2$ and $B\ D^2\ a^2 = 4\ A^2$

consequently $\frac{4\ a^2c^2 - \overline{a^2+c^2-b^2}^2}{16} = A^2$

* For Geometrical Demonstration, see Robertson's Navigation, Vol. I. p. 98; Leslie's Geometry, 6, 31, and Algebraical, Woodhouse's Trigonometry, p. 16; Simpson's Algebra, p. 264; and Leslie's Geometry, Note 49.

of the squares of any two lines or numbers their sum and difference, the last expressed thus:

$$\frac{a^2+c^2-b^2}{4} = \frac{(a+c)^2-b^2}{4} \times \frac{b^2-(a-c)^2}{4}$$

manner

$$\frac{(a+c)^2-b^2}{4} = \frac{a+c+b}{2} \times \frac{a-b+c}{2}$$

$$\frac{b^2-(a-c)^2}{4} = \frac{a+b-c}{2} + \frac{-a+b+c}{2}$$

$$\frac{a+c+b}{2} \times \frac{a-b+c}{2} \times \frac{a+b-c}{2} \times \frac{-a+b+c}{2} = A^2$$

$$\frac{a+c+b}{2} \times \frac{a+b+c}{2} - b \times \frac{a+b+c}{2} - c \times \frac{a+b+c}{2} - a = A.$$

uired the area of a triangle whose sides $276=a$,

By Natural Numbers.

6	574	574	574
37	276	387	485
185	298=S–a	187=S–b	89=S–c.
1148			
574=S			
298			
4592			
5166			
1148			
171052=S (S–a)			

Brought forward, [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

89

237880516

255893792

28468184 36=S (S—a) (S—b) (S—c)

25 53355 area = $\sqrt{S\ (S-a)\ (S-b)\ (S-c)}$

103)346

309

1063)3781

3189

10665)59284

53325

106705)595936

533525

62411

By Logarithms.

Log. 574——2.7589119

298——2.4742163

187——2.2718416

89——1.9493900

2)9.4543598

53355=4.7271799

38. *Of the Rhomboid, Trapezoid, &c.*—Every rhomboid or trapezoid which has any two sides parallel, as A B and C D, is equivalent to the sum of two triangles of equal altitude, each having one of the parallel sides for its base, and the distance between the parallels their common altitude: as, the triangles A C B and C B D are together equivalent to the whole figure A B C D, and C c is equal to B b. Whence by (Art. 37.) $\frac{C D + A B}{2} \times C c$ or B b = the area, and the rule becomes,

multiply the half of the sum of the parallel sides by the altitude (their distance), and the product is the area.

Example.—What is the area of the rhomboid whose parallel sides are 376 and 298, and the distance between the parallel sides 135?

By Natural Numbers.

376
298
674
135
3370
2022
674
2)90990
45495 the area.

By Logarithms.

376
298
674 Log.——2.8286596
135 ——2.1303338
4.9589937
2 ——0.3010300
45495 =4.6579637

39. *Of regular Polygons.*—If the centre and angular points of a polygon be joined, as A G, B G, C G, D G, E G, F G, the whole figure will be divided into as many triangles as the figure has sides, and the sum of all the triangles is equal to the whole area of the polygon A B C D E F: now the area of each triangle is the side of the polygon multiplied into the half of the perpendicular (Art. 37.); as A B,

$\frac{a\,G}{2}$=area of A B G, B C×$\frac{b\,G}{2}$=B G C, [illegible]×$\frac{c\,G}{2}$=C G D, D E×$\frac{d\,G}{2}$=D G E, E F×$\frac{e\,G}{2}$=E G F, and [illegible]×$\frac{f\,G}{2}$=A G F: but as every perpendicular from the centre upon any of the sides is equal to the radius of the inscribed circle, they are all equal to one another, and denominating the radius by r, the sum of the areas will be $\overline{A\,B+B\,C+C\,D+D\,E+E\,F+F\,A}\times\frac{r}{2}$ Hence the rule for finding the area of any polygon is, *multiply the side by the number of the sides, by half the radius of the inscribed circle, and the last product is the area.*

Example.—Required the area of a hexagon, whose side is 230, and perpendicular from the centre 199.

By Natural Numbers.	By Logarithms.
230	230——2.3617278
6	6——0.7781513
1380	99.5——1.9978231
99,5	137310 =5.1377022
6900	
12420	
12420	
137310,0 the area.	

But when the area of a regular polygon is required, there are seldom any other dimensions given more than the length of the side; therefore, to facilitate the calculation of the areas of these figures, writers upon this subject have prepared the following table of multipliers, by which the area of any polygon having its sides only given, can be easily found.

No. of Sides.		Multiplier.
3	Trig[illegible] Triangle)	0 4330127
4	Tet[illegible]	1 0000000
5	Pen[illegible]	1.7204774
6	Hexa[illegible]	2.5980762
7	Hept[illegible]	3.6339124
8	Octag[illegible]	4.8284271
9	Nona[illegible]	6.1818242
10	Decag[illegible]	7.6942088
11	Undecagon,	9.3656399
12	Duodecagon,	11.1961524

Application of the Table.—Square the value of the side, and multiply the square by the multiplier opposite the given figure, and the product will be the area.

Example.—Required the area of an octagon whose side is 325.

By Natural Numbers.

325
325
1625
650
975
105625 Square of side.
4.828427 Tabular multip.
739375
211250
422500
845000
211250
845000
422500
510002.601875 the area.

By Logarithms.

325 Log. 2.5118834
2
5.0237668
4.828 6837673
4 359
2 18
7 6
510002.6 5.7075724

40. *Of the Circle.*—If a circle be inscribed a polygon of any number of sides, and another polygon of twice the number of sides of the former be also described touching the same circle, the perimeter of the last polygon will be less than the perimeter of the first, and will touch the circle at double the number of points; and so the perimeter of every polygon will be less, and touch the circle in a greater number of points than any other polygon of less number of sides described upon the same circle; for $a\,b$, $c\,d$, $e\,f$, $g\,h$, is common to both of the perimeters of the polygons A B C D and $a\,b\,c\,d\,e\,f\,g\,h$, and $b\,c$ is less than b B+B c, and $d\,e$ is less than d C+C e, and $f\,g$ is less than f D+D g, and $a\,h$ is less than h A+A a: (for any two sides of a triangle are greater than the third) therefore $\overline{a\,b+c\,d+e\,f+g\,h}+\overline{b\,\mathrm{B}+\mathrm{B}\,c+d\,\mathrm{C}+\mathrm{C}\,e+f\,\mathrm{D}+\mathrm{D}\,g+h\,\mathrm{A}+\mathrm{A}\,a}$ is greater than $\overline{a\,b+c\,d+e\,f+g\,h}+\overline{b\,c+d\,e+f\,g+h\,a}$: but the first sum is equal to A B+B C+C D+D A, the perimeter of the polygon of four sides, and the last is equal $a\,b+b\,c+c\,d+d\,e+e\,f+f\,g+g\,h+h\,a$, the perimeter of the polygon of eight sides: consequently by doubling the number of sides continually, the difference of the perimeter of the last polygon, and the circumference of the circle, may be less than any calculable difference, or the one be ultimately equal to the other. Hence the rule for finding the area of the polygon applies to the circle, which is, *multiply the perimeter (circumference) by half the radius, and the product is the area.*

Example.—Required the area of a circle whose circumference is 722 and radius 115.

By Natural Numbers.	By Logarithms.
722 circumf.	722 ——2.8585372
575 half the	575 ——1.7596678
3610	41515 ——4.6182050
5054	
3610	
41515.0 the area	

Also, it being found that when the diameter of a circle is 1, its circumference is very nearly 3.1415926 *, and by the use of the above rule, and using 3.1416 instead of 3.1415926, we have $3.1416 \times \frac{1}{4} = .7854$ the area very near; now, because the diameter of the circle and the side of its circumscribing square are equal, and the area of the circumscribing square being 1^2 (Art. 34.): the ratio very nearly of the areas of a circle whose diameter is 1, and its circumscribing square, will be as .7854 is to 1^2; therefore if the diameter of any circle be denominated by D, the following proportion is given: as $1^2 : .7854 :: D^2$: the area of the circle, or the rule is, *multiply the square of the diameter by the decimal* .7854, *and the product is the area.*

Example.—What is the area of a circle, the diameter of which is 322.

By Natural Numbers.	By Logarithms.
322	322 Log. 2.5078559
322	2
644	5.0157118
644	.7854——$\bar{1}$.8950909
966	81433.4——4.9108027
Car. over, 103684	

* See Leslie's Geometry, 6. 32, and Robertson's Navigation, p. 103

Brot. for. 103684
.7854
414736
518420
829472
725788
81483.4136=the area.

41. *Of the Sector of a Circle.*—The area of the sector of a circle, is to the area of the whole circle in the ratio of the arc of the sector to the whole circumference: for sectors of the same circle having equal arcs are in every respect equal (Ax. 4.); therefore, if a denominate the area of the circle, c its circumference, s the area of the sector, and A B its arc, we have by proportion, $c : a :: A B : s$, or, $\frac{a \times A B}{c} = s$: but as the radius of the circle is necessarily given in the dimensions of every sector, which call r, then

$a = \frac{r \times c}{2}$ which expression of the area if substituted in the above formulæ, it becomes $\frac{\frac{r \times c}{2} \times A B}{c} = \frac{r \times A B}{2} = S$, or the rule is, *multiply the arc of the sector by half of the radius, and the product is the area.*

Example 1.—Required the area of the sector of a circle, the radius being 250 and the arc 366.

By Natural Numbers.

250
183 half of radius.
2000
250
45750 area of sector.

By Logarithms.

250 Log.——2.3979400
183 ——————2.2624511
45750=4.6603911

If the arc is given in parts of the circumference, but of a different measure from the [illegible] the radius, such as the common division of the circle into 360 parts called *degrees*, each of which is again measured by 60 subdivisions called *minutes*, and each minute into 60 parts called *seconds*, the above rule for finding the area of a sector, the arc of which is expressed in degrees, minutes and seconds, will be altered thus: *first find the area of the circle in parts of the radius, and multiply this by the number of degrees in the arc of the sector, which product divide by 360, and the quotient will be the area.*

Example 2.—What is the area of the sector, the arc of which is 50 degrees and radius 20.

By Natural Numbers.

40 diameter.
40
1600
.7854
6400
8000
12800
11200
1256,6400
50
360)62832,00(174,533$\frac{120}{360}$ area.
360
2683
2520
1632
1440
1920
1800
1200
1080
120

By Logarithms.

40——1.6020600
3.2041200
,7854——$\bar{1}$.8950909
50——1.6989700
4.7981809
360——2.5563025
174.533=2.2418784

42. *Of the Segment of a Circle.*—[illegible] the radius of the circle, then the area of the sector having the same arc as the segment by the last article: also find the area of the triangle which is formed by the chord of the segment and the radii, and subtract the last from the first when the segment is less than a semicircle, and the remainder will be the area: but the sum of these will be the area if the segment is greater than a semicircle.

Example 1.—What is the area of a segment A D B E A, its chord A B being 120, its height D E 20, and arc 128.7 *.

To find the radius.

$\frac{A\,E^2}{D\,E}$ = E F, by (Theor. 18.) and $\frac{E\,F+D\,E}{2}$ = C D the radius = 100.

128.7 the arc.
50 half of radius.
6435,0 area of sector A C B D.
4800. area of A B C.
1635 area of segment A D B E A.

120 the chord.
40 half of E C.
4800 area of triangle A B C.

Example 2.—Required the area of the opposite segment A E B F A, the chord A B being 120, its height 180, and arc A F D 499.6.

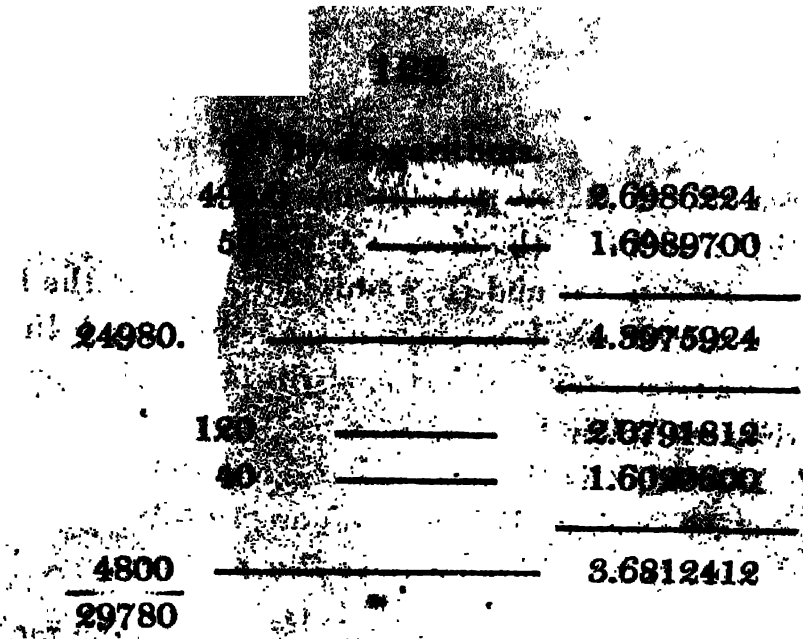
To find the radius.

$\frac{A\,E^2}{E\,F}$ = D E by (Theor. 18.) and $\frac{E\,F+D\,E}{2}$ = C E the radius = 100.

499.6 the arc.
50 half of radius.
24980,0 area of sector.
4800 area of A B C.
29780 area of segment A E B F A.

120 the chord.
40 = D C — D E.
4800 area of triangle A B C.

* I have given in these examples the dimensions of the segment only, as it is this we can alone contemplate in actual measurement, but not the radius of the circle, which is inaccessible in most cases of practice.



43. *Of the circular ring or space between two concentric circles.*—THE area of the circular ring A B C D E F, is evidently the difference of the areas of the two circles A B C and D E F, which is expressed by $\overline{BC^2\times.7854}-\overline{EF^2\times.7854}$, or $\overline{BC^2-EF^2}\times.7854$: but as $BC^2-EF^2=BC+EF\times BC-EF$ the area of the ring will be equal to $\overline{BC+EF\times BC-EF}\times.7854$; or, *multiply the sum of the two diameters by their difference, by .7854, and the last product is the area.*

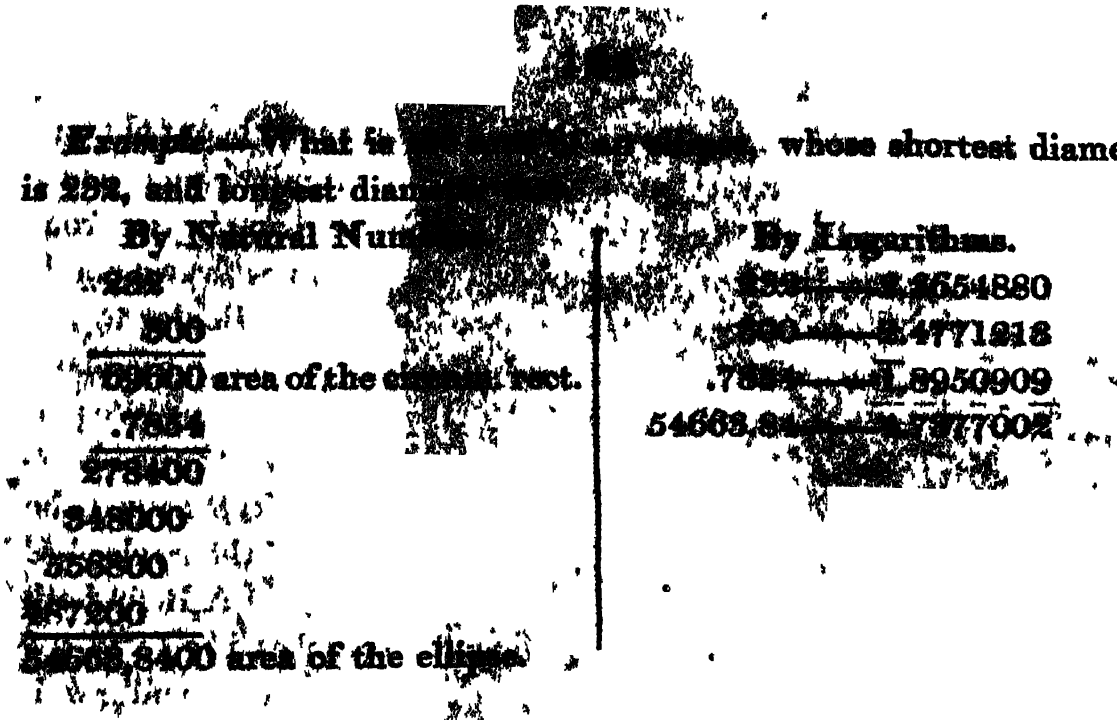
Example.—Required the area of the circular space between two circles, whose diameters respectively are 100 and 200.

200
100
300
100
30000
.7854
22562,0000 the area.

44. *Of the Ellipse.*—THE ellipse is the projection of a circle by parallel

lines drawn from one plane to another, [illegible]cal plane being [illegible] to the circular one, as the section of a [illegible] or circular roller, when cut perpendicular to its axis* is a cirrle, but oblique is an ellipse; from which it will appear, that the ellipse is described by lines drawn parallel to the axis of the cylinder upon its surface, and perpendicular to the right section or circle, and that the diameter perpendicular to the plane of inclination, or shortest diameter of the ellipse, is constantly equal to the diameter of the circle: also, every parallel drawn to the shortest diameter of the ellipse, is equal to the same projected upon the circle: for if the ellipse E G F H be the projection of the circle A C B D, upon another plane oblique to it, then E F is equal to A B, for both A B and E F are diameters of the same cylinder: also, if *e f* and *g h* are the projections of *a b* and *c d*, then *e f* is equal to *a b*, and *g h* is equal to *c d*; because *e f* passes through the cylinder at the same distance from the axis as *a b*, and *g h* the same as of *c d*, so that *e f* is the chord of the same arc as *a b*, and *g h* of the same as *c d*. (Ax. 1.) Consequently the alteration made upon a circle by this projection, can only be the elongation of its figure in the direction of the plane of inclination, and which causes the formation of the ellipse. It will appear, after the same manner, that the circumscribing square of the circle, when projected, will be a rectangle circumscribing the ellipse, having its breadth equal to the shortest diameter, and its length the longest diameter: whence we have, as the area of the square *a b c d* is to the area of the circle A C B D, so is the area of the rectangle *e f g h* to the area of the ellipse; or, as 1 : ,7854, so is the area of the rectangle to the area of the ellipse, from which proportion this rule is derived, *multiply the shortest and longest diameters together, by the decimal* .7854, *and the last product is the area of the ellipse.*

* The axis is a straight line passing through the middle of every diameter of the cylinder.



Example. What is the [illegible] whose shortest diameter is 252, and longest diam[illegible]

By Natural Numbers.

252
300
75600 area of the circumscribed rect.
.7854
275400
302800
226800
151200
54668,3400 area of the ellipse.

By Logarithms.

[illegible] 2.4014880
[illegible] 2.4771213
.7854 — 1.8950909
54668,34 — 4.7377002

45. Having now treated of the areas of simple figures, we shall, before proceeding upon the methods for finding the superficial extent of lands, notice the different measures in which this is found.

The Lineal Measures, by which the dimensions of land are taken, varies in different countries, as with us the lowest measure is an inch, from which are raised all the other greater measures, as the foot, yard, and mile. In all parts belonging to Britain the denominations are nearly the same; but the respective values vary according to the ancient standard of each country, as will appear from the following Tables.

Table of English Lineal Measure.

7.92 inch 1 link.
12 - 1½ - 1 foot.
36 - 4½ - 3 - 1 yard.
198 - 25 - 16½ - 5½ - 1 pole.
792 - 100 - 66 - 22 - 4 - 1 chain.
7920 - 1000 - 660 - 220 - 40 - 10 - 1 furlong.
63360 - 8000 - 5280 - 1760 - 320 - 80 - 8 - 1 mile.

Table of English Square Measure.

9 feet - 1 yard.
272¼ - 30¼ - 1 perch.
4356 - 484 - 16 - 1 chain.
10890 - 1210 - 40 - 2½ - 1 rood.
43560 - 4840 - 160 - 10 - 4 - 1 acre.
27878400 - 3097600 - 102400 - 6400 - 2560 - 640 - 1 mile.

Table of Scotch Lineal Measure in English Inches.

Eng. inch. 12.065 - 1 Scotch foot.
37.2 - 3⅛ - 1 Scotch ell.
223.2 - 18¾ - 6 - 1 fall.
8928.1 - 740 - 240 - 40 - 1 furlong.
71424.8 - 5920 - 1920 - 320 - 8 - 1 mile.

8.9281 Eng. inch. 1 link [illegible] chain.

892.81 or 74½ English feet } 100 - [illegible] chain. - 74 Scotch feet.

80 - - - 1 Scotch mile.

Table of Scotch Square Measure in English Inches.

Eng. inch. 79.7109 - 1 square link.

1.3065 1 square Eng. foot.

17.36 - 9.61 - 1 Scotch ell.

345.96 - 36 - 1 fall.

13838.41 - 1440 - 40 - 1 rood.

[illegible] - 5760 - 160 - 4 - 1 acre.

Table of Irish Lineal Measure in English Inches.

Eng. inch. 10.08						1 link.
12.	-	1.19	-			1 foot.
36.	-	3.57	-	3	-	1 yard.
252.	-	25.	-	21	-	7 - 1 perch.
1008.	-	100.	-	84	-	28 - 4 - 1 chain.
80640.	-	8000.	-	6720	-	2240 - 320 - 80 - 1 mile.

Table of Irish Square Measure in English Inches.

Eng. inch. 101.6064						1 link.
144.	-	1.417	-			1 foot.
1296.	-	12.755	-	9	-	1 yard.
63504.	-	625.	-	441	-	49 - 1 perch.
2540160.	-	25000.	-	17640	-	1960 - 40 - 1 rood.
10160640.	-	100000.	-	70560	-	7840 - 160 - 4 - 1 acre.

Note. Although it appears in the Table of Scotch Measure that the chain is 74 Scotch feet, yet mostly throughout all Scotland it is used only 74 English feet, but so far as I can learn there is no other authority for this reduction of the old measure, than being the custom of Land Surveyors, which likely has arisen from considering only the number of feet, and not the difference of the national measures.

Table of various standard M[illegible] Feet

The English foot is		12	000
The Paris foot		12	788
The new French foot		12	971
The Rhinland foot measured by Mr Picart		12	362
The Scots foot		12	000
The Amsterdam foot, by Snellius and Picart		11	172
The Dantzick foot, by Hevelius		11	297
The Danish foot, by Mr Picart		12	368
The Swedish foot, by the same		11	692
The Brussels foot, by the same		10	828
The Lyons foot, by Mr Auzout		13	456
The Bononian foot, by Mr Cassini		14	968
The Milan foot, by Mr Auzout		15	631
The Roman palm used by merchants, according to the same		9	791
The Roman palm used by architects		8	779
The palm of Naples, according to Mr Auzout		10	314
The English yard		36	000
The English ell		45	000
The Scots ell		37	200
The Paris aune used by mercers, according to Mr Picart		46	786
The Paris aune used by drapers, according to the same		46	680
The Lyons aune, by Mr Auzout		46	570
The Geneva [illegible]		44	760
The Amsterdam [illegible]		26	800
The Danish ell, by Mr Picart		24	930
The Swedish ell		23	380

	Inch.	Dec.
The Norway ell	24	510
The Brabant, or Antwerp ell	27	170
The Brussels ell	27	260
The Bunges ell	27	
The brace of Bononia, according to Auzout	25	
The brace used by architects in Rome	8.	7
The brace used in Rome by merchants	34	270
The Florence brace used by merchants, according to Picart	22	910
The Florence geographical brace	21	570
The vara of Seville	33	127
The vara of Madrid	39	166
The vara of Portugal	44	031
The cavedo of Portugal	27	354
The ancient Roman foot	11	632
The Persian arish, according to Mr Greaves	38	364
The shorter pike of Constantinople, according to the same	25	576
Another pike of Constantinople, according to Messrs. Mallet and De la Porte	27	920

OF CALCULATING THE SUPERFICIAL EXTENT OF LAND.

46. It has been shewn that seldom the boundaries of land can be directly measured, but is done indirectly, by station lines, and offsets; which form two or more different figures upon the ground. As the first are commonly the sides of a triangle, and the other that of a trapezoid, it is evident, the area of the field can only be found by the respective rules for these figures, when using the measured dimensions only.

Example 1.—To find the area as measured by offsets, within the boundary a A E e, and station line a F K e, in English measure.

A Triangle, (Art. 37.)	A F × a F =	15 × 10 =	150
The	A F + b g × F g =	15 + 25 × 120 =	4800
Th	b g + c h × g h =	25 + 17 × 110 =	4620
T Trapezoids,	c h + d i × h i =	17 + 30 × 70 =	3290
(by Art. 38.)	d i + E k × i K =	30 + 10 × 213 =	63900
Triangle,	E K × e K =	10 × 15 =	150
		2*)769 0	
	Acres,		.38455
			4
	Roods,		1.53820
			40
	Perches,		21.52800
			36
			316800
Area, 1 Rood, 21 Perches, 19 Yards.			158400
	Yards,		19,00800

If the line A E is curved, which is very often the case, the contents will be more than the truth if this is convex, but less when concave, towards the station lines upon which the offsets are taken, by a small segment which lieth between the boundary and a straight line joining the extremities of these upon it, as the segments A m b and C n d in this figure; whence it appears, that the nearer the offsets are taken to one another where the boundary is curved, the area derived therefrom will be nearest the truth.

* It is here the same to divide the sums of the trapezoids and triangles by 2, as taking the half of each separately, according to rules of Art 37 and 38.

Note.—Any sum of square links is easily reduced to acres, &c. as in this example of English measure, by cutting off upon the right hand 5 figures, or dividing by 100,000, the number of square links in an acre, and those upon the left will always express the acres; in the same manner, after multiplying the right hand figures by 4 for roods, 40 for perches, and 36 for yards, we shall have the left hand figures expressing the acres, roods, perches and yards; for the right hand figures are constantly the decimals of these measures, which after having explained, we need not repeat this operation in the following examples.

Example 2.—Require the area of the triangular field A B C, in Scots measure.

Find the triangle *a b c*, by the 2d Ex. of Art. 37.=70324 sq. links.

	10 + 20 × 420 =	1260
Trapezoids by Art. 38.	17 + 25 × 345 =	14490
	12 + 14 × 475 =	17100
	10 × 31 =	310
	20 × 23 =	460
Right angled triangles, by Art. 37.	17 × 27 =	459
	25 × 32 =	800
	12 × 40 =	480
	14 × 30 =	420
	2)35779 =	17889

3 Roods, 21 Falls, 4 Ells, = .88213 sq. links.

In the above example there are some dimensions more than merely for constructing the figure, as shown by the additional measurement of 30. 31. 23. 27. 32. 40. which are necessary for finding the area of the small spaces left out on the angles by the offsets, and which

more than two, these form the sides of one or more great triangles, from which, offsets to the angular points of the field, are measured perpendicular by a cross staff.—This instrument I have already shewn should not be used for long lines, when any degree of accurment required, and it would be quite unnecessary to give the following example, were this not to shew a bad practice.

Example 5.—Find the area of each of the three fields, within A *h i k* B *l* from the following dimensions, made upon only one leading line A B.

	A *c* = 140		*c h* = 345
	c d = 325		*d l* = 326
	d e = 40	Offsets.	*e i* = 403
A B = 921.	*e n* = 12		*f m* = 26
	n f = 24		*g k* = 318
	f g = 264		
	g B = 116		

To find the Area of the field A *h m l.*

A *c* × *c h* = 140 × 345
c f × *c h* = 401 × 345
} 541 × 345* = 186645

c n × *f m* = 377 × 26 = 9802

A *n* × *d l* = 517 × 226 = 116842

2)313289

Area of A *h m l*, 1 acre, 2 roods, 10 perches, 22.7 yards. 1.56644

* When two unequal quantities are to be multiplied by the same quantity, as 140 and 401 by 345, the result desired will be found by adding the two unequal quantities together, and multiplying by the common multiplier.

To find the Area of the field *l m k* B.

n B × *n l* = 304	× 226 —	91304
n g × *f n* = 288	× 26 —	7488
f g × *g k* = 264	× 318 —	83952
g B × *g k* = 116	× 318 —	36888
		2)219632

Area of *l m k* B, = 1 acre, 15 perches, 25.4 yards. 1.09816

To find the Area of the field *m h i k.*

e B + *e i* × *c e* = 748	× 365 —	273020
e i + *g k* × *e g* = 721	× 300 —	216300
Subtract		489320
c h + *f m* × *c f* = 371	× 401 —	148771
f m + *g k* × *f g* = 344	× 264 —	90816
		239587
		2)249733

Area of *m h i k*, = 1 acre, 39 perches, 28.3 yards. 1.24866

	A.	R.	P.	Y.
And the collective Area	1	2	10	22.7
of the three fields is	1	2	15	25.4
	1	0	39	28.3
	3	3	26	4.4

By the above method it is almost constantly necessary to produce the perpendiculars across one, and sometimes two fields to the opposite angle, which by reason of the imperfections of the cross staff, renders such measurements quite unfit for constructing a correct plan; besides it can only be used on open and very level lands; for if otherwise, there will frequently happen interruptions to the measurements, if not made wholly impracticable, of those offsets where most desired, from intervening trees, houses, and swelling or sloping grounds, likewise by which the point of intersections upon the leading line cannot

be directly found. Also whatever curvature is upon the boundary between the perpendiculars, and as this is concave or convex towards the leading line, will cause an error of deficiency or excess by the segment contained upon it; so that by this method all the boundaries would require to be straight lines between the perpendiculars; for it cannot be expected of those who are in this practice, can take the time of making the offsets so near each other to avoid this error, as can so easily be done, when the station line is run near the boundary. But with all these disadvantages, I have no doubt there are practitioners who may set forth ways and means of overcoming, or at least presume to render such operations sufficiently accurate, however, unscientific, and slovenly it may appear to others *.

48. Before we can give a rule for finding accurately the respective areas of two or more fields together, from a series of triangles covering the lands, and the dimensions of which will also construct the plan as shewn in the last section, it is necessary to demonstrate the following theorem, which I have reserved for this place, as particularly applicable for the purpose.

Theorem—Triangles which have a common angle, are to each other as the rectangles of their containing sides.

For let A B C, and D B E be two triangles, having the same common angle at B, and denominate A B = a A C = b D A = c A E = d and the perdendicular E E = p,

then we have by Theor. (9). $\frac{a \times p}{2}$ = area A B E

$\frac{c \times p}{2}$ = area A D E

* Those who wish to see several examples by the cross staff, may consult Ainslie's Landsurveying, page 98, Crocker, Nisbet, &c.

therefore area A B E : area B D E : : a : c

in the same manner B D C : area B D E : : b : d

and area A B E : area B D E : : $a\,d$: $c\,d$

area B D C : area B D E : : $c\,b$: $c\,d$

wherefore area A B E : area B D C : : $a\,d$: $c\,b$

and consequently must area A B C : area B D E : $a\,b$; $c\,d$

Cor. 1.—Hence triangles having one common angle are as the difference of the squares of the half sum, and half difference of the containing sides, for $a\,b = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$.

Cor. 2.—Also similar triangles are as the square of their homologous sides.

for A B C : A D E : $a\,b$: : $c\,d$

but a : c as b : d for D E and B C are parallel. (Theor. 16.)

hence A B C : A D E : : a^2 : c^2

Cor. 3.—And all rectilineal figures are as the squares of their homologous sides: for dividing these each into similar triangles, viz. A, B, C, D, E, and A′, B′, C′, D′, E′; and calling the homologous side of each triangle, a, b, c, d, e, and a', b', c', d', e' : by Cor. 2.

A : A′ : a^2 : a'^2

B : B′ : b^2 : b'^2

C : C′ : c^2 : c'^2

D : D′ : d^2 : d'^2

E : E′ : e^2 : e'^2

and A : B : : a^2 : b^2

A : C : : a^2 : c^2

A : D : a^2 : d^2

A : E : a^2 : e^2

wherefore A : a^2 : : A + B + C + D + E : $a^2 + b^2 + c^2 + d^2 + e^2$

and $A+B+C+D+E : A+B+C+D+E :: a^2+b^2+c^2+d^2+e^2 : a'^2+b'^2+c'^2+d'^2+e'^2$

and consequently $A+B+C+D+E : A+B+C+D+E : a^2 : a'^2$

$$\text{——————————} : b^2 : b'^2$$

$$\text{——————————} : c^2 : c'^2$$

$$\text{——————————} : d^2 : d'^2$$

$$\text{——————————} : e^2 : e'^2$$

By the Theorem we have the following Rule for finding the angular portion of the area of any triangle, which is divided into two parts by a straight line, when the distance of the intersecting points of this line, upon the two sides from the angular point are known *Multiply the area of the whole triangle by the distances of the intersecting line upon the sides from the angular point, the product of which divide by the product of the sides, and the quotient will be the area of the angular portion.*

Example 7.—Calculate the respective areas of the two Fields, A and B.

To find the triangle *a b c.*

$\frac{646+358+663}{2} = 833.5$ —— 2.9209056

Difference of $a\ c = 187.5$ —— 2.2730013

—————— of $b\ c = 475.5$ —— 2.6771505

—————— of $a\ c = 170.5$ —— 2.2317244

2)10.1027818

Log. of area = 5.0513909 = 112561

To find the triangle *a c d.*

$\frac{663+572+481}{2}$ = 858 —— 2.9334873

Difference of $a\ c$ = 195 —— 2.2900346

—— of $d\ c$ = 286 —— 2.4563660

—— of $a\ d$ = 377 —— 2.5763414

2)10.2562293

Log. of area, 5.1281146 = 134312

To find the angular portions $a\ e\ f$ and $c\ f\ g$.

Log. area of $a\ b\ c$ as above, —— 5.0513909

Distance $a\ e$ = 276 —— 2.4409091

Distance $a\ f$ 333 —— 2.5224442

10.0147442

Side $a\ b$ = 646 —— 2.8102325

Side $a\ c$ = 663 —— 2.8215135

—— 5.6317460

4.3829982 = 24154

Log. area of $a\ c\ d$ as above is 5.1281146

Distance $c\ f$ = 230 —— 2.5185139

Distance $c\ g$ = 282 —— 2.4502491

10.0968776

Side $c\ a$ = 663 —— 2.8215135

Side $c\ d$ = 572 —— 2.7573960

—— 5.5789095

4.5179681 = 32958

By inspection of the figure, $a\ e\ g\ d$ = area $a\ e\ f$ + area $a\ b\ c$ — area $c\ f\ g$.

and as above, area $a\ e\ f$ = 24154
area $a\ c\ d$ = 134312
158466
Subtract area $c\ f\ g$ = 32958
Area $a\ e\ g\ d$ = ——— 125508

	(276 × 6 + 7)	= 3588
	(10 × 6 + 7)	= 120
Add Trapezoids,	(481 × 10 + 10)	= 9620
	(10 × 8 + 8)	= 160
	(572 — 282 × 8 + 10)	= 5220
		2)18708
		9354

Total area of Field A, = 1.34862 = 1..1..15,7

In the same manner $e\ b\ c\ g$ = area $c\ f\ g$ + area $a\ b\ c$ — area $a\ e\ f$.

and as above,	area $c\ f\ g$	= 32958
	area $a\ b\ c$	= 112561
		145519
	Subtract $a\ e\ f$	= 24154
	area of $a\ b\ c\ g$	= 121365

	(282 × 10 + 12)	= 4816
	(12 × 6 + 10)	= 192
Add Trapezoids,	(358 × 10 + 10)	= 7160
	(10 × 9 + 10)	= 190
	(642 — 276 × 9 + 7)	= 5920
		2)18278
		9139

Total area of Field B, = 1.30504 = 1..1..8,8

After the same manner the respective areas of three fields A, B, C, may be calculated, as Field A by the figure is equal to $a\,e\,f + a\,c\,d - f\,c\,i$, Field B $= a\,g\,h + f\,c\,i - a\,e\,f - c\,h\,k$; and Field C $= c\,h\,k + a\,b\,c - a\,g\,h$; after adding the offsets belonging to each as shewn above.

Where it has been necessary to measure between any two of the sides of a triangle as $f\,i$, it is obvious that the area of the triangle $f\,c\,i$ can be found from its three sides now given, instead of using the above rule.

There are cases in which it may be necessary to take some dimensions besides primary triangles; but these will vary according to the relative position of the fences with the sides or station lines, as in the three fields D, E, F, where the measurements $g\,f$, $g\,k$, $f\,h$, are additional for finding the area of the small triangle $g\,h\,f$. as

Field D $= a\,b\,i + a\,i\,a + g\,h\,f - e\,l\,k - c\,i\,f$

Field E $= c\,i\,b + c\,i\,f + c\,h\,k - g\,h\,f$

Field F $= l\,e\,h + c\,e\,d - c\,h\,k$

When besides the primary triangles, other lines within these are likewise measured for delineating the boundaries of a number of lots, areas or buildings, by which the triangles are divided into irregular polygons; the above rule in this case is not applicable for finding the respective area of each, without such be resolved into triangles, and the additional measurements as last shewn are also made, whereby the whole figure is divided into parts of the triangle, fit to be calculated as above. For this purpose it is not always necessary to make these, actually upon the ground, but upon the plan after delineated by the field dimensions, when such additional lines may be drawn and measured by the scale, which with those actually measured upon the ground, the respective areas can be found.

49.—Hertofore I have only shewn the methods of calculation of areas by Natural Numbers and Logarithms; but as the first causes a multiplicity of figures both tedious and liable to error, and the other is not applicable for adding the products of quantities, which is a continual operation in the calculation of areas. I have here following, given another method by the use of a Table of Square Numbers, which is peculiarly applicable to the last purpose, and requiring only in the actual operation for finding the sums of the products of any number of pairs of multipliers, the addition and subtraction of two sums. For facilitating this method of calculation, and as particularly useful to Landsurveyors, I have calculated the following Table of Square Numbers from 1 to 2000.

Table of Square Numbers from 1 to 2000.

N	0	1	2	3	4	5	6	7	8	9
1	100	121	[illegible]	[illegible]	196	225	256	289	324	361
2	400	441	[illegible]	[illegible]	576	625	676	729	784	841
3	900	961	1024	1089	1156	1225	1296	1369	1444	1521
4	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
5	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
6	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
7	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
8	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
9	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801
10	10000	10201	10404	10609	10816	11025	11236	11449	11664	11881
11	12100	12321	12544	12769	12996	13225	13456	13689	13924	14161
12	14400	14641	14884	15129	15376	15625	15876	16129	16384	16641
13	16900	17161	17424	17689	17956	18225	18496	18769	19044	19321
14	19600	19881	20164	20449	20736	21025	21316	21609	21904	22201
15	22500	22801	23104	23409	23716	24025	24336	24649	24964	25281
16	25600	25921	26244	26569	26896	27225	27556	27889	28224	28561
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199	3960100	3964081	3968064	3972049	3976036	[illegible]	[illegible]16	3988009	3992004	3996001

Application of the Table.

If it is required to find the square of any number below 200, as 169, this is always found in col. 8, as 28561, after rejecting the two cyphers upon the right hand—also the same is found opposite No. 16, in col. 9, and of every number below 2000 the square is found in the column of the number of the right hand figure, and opposite the number expressed by the remaining figures upon the left, as the square of 1786, in col 6, opposite No. 178 is 3189796. But if a decimal is attached to the number required, as 1786.3, the following rule may be used. Add to the square of the integral number, the tenth part of the product of the integral number and twice the decimal, also to which add as a decimal the square of the decimal, as

the square of 1786 by the Table is 3189796

1786×6 1071.6

.3 squared - .09

square of 17.63 = 3190867.69

If 17863 be required, the operation becomes the same, with the exception only of the decimal point which is now rejected, as 319086769 will be the square;—whence it appears, with the use of the above tables and rule, we always can find the square of any number below 20000.

Example 1.—Required by the table of square numbers to find the product of $a\ b$ $a = 348$ and $b = 236$.

By Cor. 1. of (Art. 48.) $a\ b = \left(\frac{a \times b}{2}\right)^2 - \left(\frac{a - b}{2}\right)^2$ wherefore the operation of this example.

$\left(\frac{a+b}{2}\right)^2 = \left\{\begin{matrix}348\\236\end{matrix}\right\}\begin{matrix}584\\2\end{matrix} = 292$ the square of, by Table $= 85264$

$\left(\frac{a-b}{2}\right)^2 = \frac{112}{2} = 56$ the square of, by Table $= \underline{3136}$

the difference, - - $\overline{82128}$ and product of 348×236 as required.

Example 2.—Required by the Table, the sum of the products of 247×245, 236×458, 369×169, 235×174.

Multipher.	Sums.	Half Sums.	Squares.	Differences.	Half Differences.	Squares.
247 245	492	= 246 -	60516	2 -	1 -	1
236 458	694	= 347 -	120409	222 -	111 -	12321
369 169	538	= 269 -	72361	200 -	100 -	10000
235 174	409	= 204,5 -	41820,25	61 -	30,5 -	930,25
			295106			23252
Subtract			23252			
Sum of products required,			271854			

It appears in the above, when the sum is odd, as 409, the difference must likewise be odd, and the decimal .25 is produced equally on both sides, which therefore may be rejected in the additions of the operation. By taking the squares of the sums and differences, instead of the half sums and half differences, and dividing the difference of the sums of these squares by 4, will shorten the above operation: or by using both, the one may be the proof of the other *, as

* It is satisfactory to observe, that the greatest confidence may be placed in the above table, as these have been fully proven upon the proof sheet from the press, so that the proof only now required, is upon the operation of addition and extracting these squares from the tables.

Multipliers.	Sums.		Differences.	Squares.
247 } 245 }	492	[illegible]	2	4
236 } 458 }	694	[illegible]	222	49284
369 } 169 }	538	[illegible]	200	40000
235 } 174 }	409	[illegible]07281	61	3721
		1180425		93009
		93009		
		4)1087416		
The sum of the products as before.		271854		

50.—As the dimensions for constructing a plan may be always fewer than those for calculating the area of lands, from this it is the common practice in extensive surveys, to take such dimensions that are only necessary for planning, and afterwards by the scale from which the delineation has been made to find new dimensions, whereby the superficial area is more conveniently calculated than by the first. It is evident, by this method, the area of the superficial extent depends wholly upon the accuracy of the delineation made by the first dimensions; the manner of taking which, has been sufficiently shewn under the last section, so far as regarding surveys to be made with the chain only.

A common method of calculating [illegible] from the delineation, is by dividing each field into triangles [illegible] the scale measuring the base and perpendicular of each [illegible] taking the collective contents for the area of that part of [illegible] which these cover; as in the field A B C D, measure by the [illegible] base A B, and perpendicular C C, and base A C and perpendicular [illegible] D' which after being calculated according to Art. 37, gives the [illegible] of the whole field.

When one or more of the sides of the field is curved or crooked, as the side A B, it is necessary to draw a straight line, making equal areas on both sides between it and the crooked line and calculating the field as above, after having taken the dimensions upon this straight line as its boundary.

The manner of drawing a straight line so as to contain equal areas on both sides of a curved or crooked line is sometimes by surveyors in practice, done by judging with the eye, till the space upon each side of it appear equal; but where accuracy is desired, the following method should be used.

If it is required to straight the crooked line A B C by drawing A D from the point A, to meet another line C D which is at any given angle to B C; first draw through B, the straight line B D parallel to A C, and meeting C D in D; then join A D, which will be the straight line required; for the triangles A B C and A D C are equivalent by Cor. 2. Theor. 9.

This problem may be applied generally, and to any crooked line whatever; as let 8. 7, 6, 5, 4, 3, 2, 1, I, be one of the sides of a field which is required to be straight by a line drawn from the point 8, and to cut the next side, the direction of which is I K. First cut I K, by a line through 1 parallel to 2 I, and mark this intersection 1, next cut I K, by drawing a line through 2; and a parallel to 3. 1, and mark this intersection 2, again through 3, and parallel to 4. 2 cut I K, and mark the intersection 3; also make the

intersection 4, through 5, [illegible] to 6. 3; and 5 through 6, parallel to 7. 4; and 6 th[illegible] parallel to 8. 5; and lastly join 8. 6, which will be the line [illegible]

If the line represent[illegible] A B C D E F, it becomes first necessary in this case to [illegible] into a certain number of straight lines by the eye, but each as nearly coinciding with the curve, as to avoid any sensible error [illegible] A B, B C, C D, D E, E F, and afterwards resolving those into one straight line A H, by the above method.

51. The second method of calculating from the delineation is by drawing parallel lines at equal distances from each other covering the whole field, and upon the extremities of each parallel space, straightening the boundary by a perpendicular to the parallels; then taking the lengths of each space, and adding them together, the sum of which if multiplied by the common breadth will give the area; as let the area of the field A B C D be required.

In laying off the parallels, it will save the trouble of straightening upon one of the sides, if these are drawn perpendicular to that which is a straight line, as D B, by which the side A C, is only necessary to be straightened upon each parallel space, by lines drawn parallel to DB; but as it is common that the angles made by the fences are not often right angles, there will be angular portions both taken in and left out by the outermost parallel lines, which must be calculated separately, and added or subtracted to the collective area of the parallel spaces as the position of such requires, as the triangle C D D′ must be subtracted, and triangle A B B′ is to be added; as

* By thus numbering the angles both upon the crooked line, and I K, the error of not drawing through the two angles is easily avoided, for this is always the number lying between the number upon I K, and that upon the crooked line, as the line drawn through 4 is parallel to 5 and 3, and the intersection of this is also 4 upon I K

	284
	[illegible]
	[illegible]
	412
	330
	[illegible]60
	[illegible]00
the common distance,	100
	250000
Add A B B′ $\frac{460 \times 12}{2}$	2760
	252760
Subtract C D D′ $\frac{376 \times 26}{2}$	4888
Area of A B C D, =	247872

If it is found after the parallel lines are drawn that the boundary is a straight line between each. The following formulæ may be used for calculating the area of the parallel spaces, where *a*, *b*, *c*, *d*, *e*, *f*, denominates the respective lengths of the parallel lines, D their common distance, and A the collective area.

$$A = \left(\frac{a+f}{2} + b + c + d + e\right) D$$

as

a	245
f	284
	2)529
	264.5
b	272
c	236
d	306
e	270
	1348.5
D	100
Area, =	134850.0

If the boundary is a [illegible] curve, and the number of the parallel lines [illegible] formula may be adopted without straightening the [illegible]

$$\text{Area} = \frac{a + 4b + 2(c + e)}{} \times D$$

or calling the sum of [illegible], the sum of the second, fourth, sixth, &c. B, and [illegible] rest C.

$$\text{Area} = \frac{A + 4B + 2C}{3} \times D,$$

putting D for the common distance as before *, as

Required the area of the field A B C D, measured by equidistant parallels.

	112	120	118
	136	116	12^2
A =	248	128	140
4 B =	1456	364 = B	380 = C
2 C =	760	4	2
	3)2464	1456 = 4 B	760 = 2C
	821.33		
D - -	100		
Area =	82133,00		

52. One of the best methods of calculating areas from the plan is after straightening the irregular boundaries of any figure, as A B C D E is to draw lines through every angle parallel to one of the sides, as B B D D′ C C′ all parallell to A E, and afterwards intersect these by a perpendicular line, either within or without the figure: then by measuring the lengths of each parallel line within the figure, as A E 356,

* This is not a perfect rule but a very near approximation. See Dr Hutton's Mensuration, page 374

B B′ = 563, D D′ = 296, and on the perpendicular their distance $a\,b = 246$, $b\,c = 96$, $c\,d = 32$ [illegible] the dimensions of the trapezoids A B B′ E, and B D D′ [illegible] angle D C D′, which together comprehends the area of the [illegible] figure A B C D E, and if calculated by Art. 38 stand as follows:

356 } 563 }	919 × 246	[illegible]074
563 } 296 }	859 × 96	82464
	296 × 32	9472
		1318010
Total Area,		15900 5

This method has considerable advantage in practice, over the methods of triangles, by the facility of proving the dimensions, and of equal parallel lines in having fewer; as the sum of the heights of the trapezoids, and the whole line $a\,d$, should be exactly equal; next by taking the half length of the first measurement of each parallel line from the scale, this should step exactly twice its respective length, which together affords a complete proof of all the scale dimensions. Whereas in the method of triangles the perpendiculars of each are upon their respective bases, and may be proven by a repetition of the same measurements, but without a proof in a collective sum as in the above.

It will appear obvious in the calculation of areas by the scale, the fewer dimensions which can be used, and these proven collectively instead of separately, that the results will be nearest the truth; for there are two sources of error when calculating by the scale in the way of perfect agreement with the true content as would be derived, if possible to calculate the same from the dimensions by which the plan has

been constructed; th[illegible] inaccuracy arising in taking off the lengths of the [illegible] out for calculation, and the other are errors, ar[illegible] not being drawn exactly from the centre of the [illegible] the extremities, by which the measurements on th[illegible] may be less, and upon the other greater than if the cont[illegible] first of these means of error may be overcome by proving [illegible] the perpendiculars collectively with an equal divided scale; but the other will always arise to a degree, and especially the more lines which are employed in the calculations.

Besides the means of error above stated, there is a third peculiar to calculations made upon paper, which is the expansive or contracting power of this substance by moisture or dryness, which may alter the whole surface of the plan less or more as it is exposed to the extremes of either of those states of the atmosphere; which circumstance points out to all surveyors the necessity of having an office not in the least liable to damp, but which will be in nearly an equal state of dryness throughout the year. This also shews the necessity of having one line marked out to the whole length of the same scale by which the plan is delineated, and upon the same sheet, to which an exact reference of the expansion or diminution above or below the original scale may be always made.

53. In Art. 4, I pointed out the method of finding the *chain error*, and now answerable to which I have calculated the following Table for finding that correction in superficial measure for every acre.

TABLE

Shewing the Superficial Correction upon every Acre of the Chain error from one-tenth of an inch to four [illegible], either below or above the length of the Scots or English Chain.

SCOTS MEASURE.

Length of the Chain in Feet.		Length of the Chain in Links.		Sup. Correct. of every Acre.
F.	I.	L.	Dec.	Falls.
74	0.1	100	.011	= .035
74	0.2	100	.022	= .070
74	0.3	100	.033	= .105
74	0.4	100	.045	= .144
74	0.5	100	.056	= .179
74	0.6	100	.067	= .214
74	0.7	100	.078	= .249
74	0.8	100	.090	= .288
74	0.9	100	.101	= .323
74	1.0	100	.112	= .358
74	1.1	100	.123	= .393
74	1.2	100	.135	= .432
74	1.3	100	.146	= .467
74	1.4	100	.157	= .502
74	1.5	100	.168	= .538
74	1.6	100	.180	= .576
74	1.7	100	.191	= .611
74	1.8	100	.202	= .647
74	1.9	100	.213	= .682
74	2.0	100	.225	= .720
74	2.1	100	.236	= .756
74	2.2	100	.247	= .791
74	2.3	100	.259	= .829
74	2.4	100	.270	= .865
74	2.5	100	.281	= .900
74	2.6	100	.292	= .935
74	2.7	100	.304	= .971
74	2.8	100	.315	= 1.009
74	2.9	100	.326	= 1.044
74	3.0	100	.337	= 1.080
74	3.1	100	.349	= 1.118
74	3.2	100	.360	= 1.151
74	3.3	100	.371	= 1.189
74	3.4	100	.382	= 1.224
74	3.5	100	.394	= 1.262
74	3.6	100	.405	= 1.298
74	3.7	100	.416	= 1.335
74	3.8	100	.427	= 1.369
74	3.9	100	.438	= 1.407
74	4.0	100	.450	= 1.443
74	4.1	100	.461	= 1.478
74	4.2	100	.472	= 1.513
74	4.3	100	.48	= 1.552
74	4.4	100	.49	= 1.587

ENGLISH MEASURE.

Length of the Chain in Feet.		Length of the Chain in Links.		Sup. Correct. of every Acre.
F.	I.	L.	Dec.	Perches.
66	0.1	100	.012	= .038
66	0.2	100	.025	= .080
66	0.3	100	.037	= .118
66	0.4	100	.050	= .160
66	0.5	100	.063	= .201
66	0.6	100	.075	= .240
66	0.7	100	.088	= .281
66	0.8	100	.101	= .323
66	0.9	100	.113	= .361
66	1.0	100	.126	= .402
66	1.1	100	.137	= .438
66	1.2	100	.151	= .483
66	1.3	100	.164	= .524
66	1.4	100	.176	= .562
66	1.5	100	.188	= .601
66	1.6	100	.202	= .645
66	1.7	100	.214	= .684
66	1.8	100	.227	= .725
66	1.9	100	.239	= .763
66	2.0	100	.253	= .808
66	2.1	100	.265	= .846
66	2.2	100	.277	= .885
66	2.3	100	.290	= .926
66	2.4	100	.30	= .971
66	2.5	100	.315	= 1.006
66	2.6	100	.328	= 1.047
66	2.7	100	.340	= 1.086
66	2.8	100	.353	= 1.127
66	2.9	100	.366	= 1.169
66	3.0	100	.378	= 1.207
66	3.1	100	.391	= 1.248
66	3.2	100	.404	= 1.290
66	3.3	100	.416	= 1.328
66	3.4	100	.429	= 1.369
66	3.5	100	.441	= 1.408
66	3.6	100	.454	= 1.449
66	3.7	100	.467	= 1.491
66	3.8	100	.479	= 1.529
66	3.9	100	.492	= 1.570
66	4.0	100	.505	= 1.612
66	4.1	100	.517	= 1.650
66	4.2	100	.530	= 1.691
66	4.3	100	.542	= 1.729
66	4.4	100	.555	= 1.771

Application of the Tables.

The first colum[illegible] Scotch Measure and English Measure, expre[illegible] plus, in inches and tenth parts of an inch, of t[illegible] of the chain; the second columns is the links a[illegible] answering to the first; the third columns contain[illegible] rection in superficial measure expressed in falls or perches to be [illegible] upon every acre, answering either to the second columns; and the [illegible] three columns of each Table is the same but minus of the respective lengths of the chain.

Example.—Required the true extent, of which the calculated contents are 46 : 3 : 10, from measurement made with a chain 74 feet 2 inches in length in Scots Measure.

Calculated contents, - - -	A. R. F. 46 : 3 : 10
Superficial correction by the Table of Scots Measure for the chain error of 2 inches plus 74 feet upon every acre is 1.044 falls, which calculated by proportion for 46 : 3 : 10— is - - -	0 : 1 : 8.87
True extent, - - -	47 : 0 : 18.87

Example 2d.—Required the true extent, of which the calculated contents is A. R. F. 23 : 3 : 35 from measurements made with a chain $1\frac{8}{10}$ inches minus of 66 feet in English Measure.

Calculated contents, - - -	A. R. F. 23 : 3 : 35
Superficial correction by the Table of English Measure for chain error $1\frac{8}{10}$ inches minus of 66 feet, is upon every acre, .482 fall which calculated by proportion for A. R. F. 23 : 3 : 35 — is	0 : 0 : 11.55
True extent,	23 : 3 : 23.45

In the case where the chain error is greater than in the above Table, the following formulæ may be used for calculating the superficial correction, after denominating the standard length of the chain by a, the chain error by b, and $\mp C$, the superficial correction, as the chain error is below or above the standard.

$$-C = 20\,a\,b + 10\,b^2$$
$$+C = 20\,a\,b - 10\,b^2$$

Example. Find the superficial correction for the chain error of 5 inches, or .518 links plus, Scots measure.

a = 100 links.
20
2000
b = .518
1036.000
$10\,b^2$ = 2.683
1038.683 superficial correction in square links to be subtracted for every acre.

If the same chain error be minus, then

a = 100 links.
20
2000
b = .518
1036.000
$10\,b^2$ = 2.683
1033.317 superficial correction in square links to be added for every acre *.

* Observe in laying out ground with a short Chain or error minus, as $74\frac{F.}{-}\frac{I.}{2\,2}_{75}$, the quantity wanting of the true area is of the same sign of the Chain error, as in this, 1.041 falls, or $20\,a\,b - 10\,b^2$, and if the contrary or $75\frac{I.}{+}\frac{2}{75}$ the excess land off will be 1.044 or $20\,a\,b + 10\,b^2$.

In the foregoing when giving the rules for the calculation of areas, I omitted purposely those [illegible] the areas from the sides and angles together, but which [illegible] fully exemplified under Trigonometry.

53. The following [illegible] are such which their respective titles explain, and are of constant use to the practical Surveyor.

TABLE,

Shewing the corresponding Square Links, in any number of Roods, and Perches or Falls, from one Fall or Perch to one Acre.

Acre. Roods. Perches or Falls.	Square Links.	Acre. Roods. Perches or Falls.	Square Links.	Acre. Roods. Perches or Falls.	Square Links.	Acre. Roods. Perches or Falls.	Square Links.
1	625	[illegible]	[illegible]	2.. 1	50625	3.. 1	75625
2	1250	[illegible]	[illegible]	2.. 2	51250	3.. 2	76250
3	1875	[illegible]	[illegible]	2.. 3	51875	3 3	76875
4	2500	[illegible]	[illegible]	2.. 4	52500	3.. 4	77500
5	3125	[illegible]	[illegible]	2.. 5	53125	3.. 5	78125
6	3750	[illegible]	[illegible]	2.. 6	53750	3.. 6	78750
7	4375	[illegible]	[illegible]	2.. 7	54375	3 7	79375
8	5000	[illegible]	[illegible]	2.. 8	55000	3.. 8	80000
9	5625	[illegible]	[illegible]	2.. 9	55625	3 . 9	80625
10	6250	[illegible]	[illegible]	2..10	56250	3 10	81250
11	6875	[illegible]	[illegible]	2..11	56875	3..11	81875
12	7500	[illegible]	[illegible]	2..12	57500	3 12	82500
13	8125	[illegible]	[illegible]	2..13	58125	3 13	83125
14	8750	[illegible]	[illegible]	2..14	58750	3 14	83750
15	9375	[illegible]	[illegible]	2..15	59375	3 15	84375
16	10000	1 16	35000	2..16	60000	3 16	85000
17	10625	1..17	35625	2..17	60625	3..17	85625
18	11250	1..18	36250	2..18	61250	3 18	86250
19	11875	1 19	36875	2..19	61875	3..19	86875
20	12500	1..20	37500	2..20	62500	3..20	87500
21	13125	1 21	38125	2..21	63125	3 21	88125
22	13750	1..22	38750	2..22	63750	3 22	88750
23	14375	1 23	39375	2..23	64375	3 23	89375
24	15000	1..24	40000	2..24	65000	3..24	90000
25	15625	1..25	40625	2..25	65625	3 25	90625
26	16250	1 26	41250	2..26	66250	3 26	91250
27	16875	1..27	41875	2..27	66875	3 27	91875
28	17500	1..28	42500	2..28	67500	3 28	92500
29	18125	1..29	43125	2 29	68125	3 29	93125
30	18750	1..30	43750	2..30	68750	3 30	93750
31	19375	1 31	44375	2..31	69375	3 31	94375
32	20000	1..32	45000	2..32	70000	3..32	95000
33	20625	1 33	45625	2 33	70625	3..33	95625
34	21250	1 34	46250	2 34	71250	3 34	96250
35	21875	1 35	46875	2 35	71875	3 35	96875
36	22500	1..36	47500	2 36	72500	3 36	97500
37	23125	1 37	48125	2 37	7312	3 37	981
38	23750	1..38	48750	2 38	73750	3..38	987
39	24375	1..39	49375	2..39	74375	3 39	9937
1.. 0	25000	2 . 0	50000	3.. 0	75000	0 0	100000

TABLE,

Shewing the number of English Acres, answering to any number of Scots, from 1 to 100.

Scots Acres.	English Acres.	Scots Acres.	English Acres.
A.	A. R. P.		A. R. P.
1	1..1.. 1.13	51	64..0..18.08
2	2..2.. 2.27	52	65..1..19.17
3	3..3.. 3.41	53	66..2..20.31
4	5..0.. 4.55	54	67..3..21.45
5	6..1.. 5.69	55	69..0..22.59
6	7..2.. 6 82	56	70..1..23.72
7	8..3.. 7.96	57	71..2..24.86
8	10..0.. 9.10	58	72..3..26.00
9	11..1..10.24	59	74..0..27.14
10	12..2..11.38	60	75..1..28.28
11	13..3..12.51	61	76..2..29.41
12	15..0..13.65	62	77..3..30.55
13	16..1..14.79	63	79..0..31.69
14	17..2..15.93	64	80..1..32.83
15	18..3..17.07	65	81..2..33.97
16	20..0..18.20	66	82..3..35.10
17	21..1..19.34	67	84..0..36.24
18	22..2..20 48	68	85..1..37.38
19	23..3..21.62	69	86..2..38.52
20	25..0..22.76	70	87..3..39.66
21	26..1..23.89	71	89..1.. 0.79
22	27..2..25 03	72	90..2.. 1.93
23	28..3..26.17	73	91..3.. 3.07
24	30..0..27.31	74	93..0.. 4.21
25	31..1..28.45	75	94..1.. 5.35
26	32..2..29.58	76	95..2.. 6.46
27	33..3..30.72	77	96..3.. 7.62
28	35..0..31.86	78	98..0.. 8.76
29	36..1..33.00	79	99..1.. 9.90
30	37 .2..34.14	80	100..2..11.04
31	38..3..35.27	81	101..3..12.17
32	40..0..36.41	82	103..0..13.31
33	41..1..37.55	83	104..1..14.45
34	42..2..38.69	84	105..2..15.59
35	43..3. 39.83	85	106..3..16.73
36	45..1.. 0.96	86	108..0..17.86
37	46..2.. 2.10	87	109..1..19.00
38	47..3.. 3.24	88	110..2..20.14
39	49..0.. 4 38	89	111..3..21.28
40	50..1.. 5 52	90	113..0..22.42
41	51..2.. 6.65	91	114..1..23.55
42	52..3.. 7.79	92	115..2..24.69
43	54..0.. 8.93	93	116..3..25.83
44	55. 1..10.07	94	118. 0..26.97
45	56..2..11.21	95	119..1..28.11
46	57..3..12.34	96	120..2..29.24
47	59 .0 .13.48	97	121..3..30.38
48	60..1..14.62	98	123..0..31.52
49	61..2..15.76	99	124 1..32.66
50	62..3..16.90	100	125..2..33.80

TABLE,

Shewing the number of Scots Acres, answering to any number of English Acres, from 1 to 100.

Engl. Acres.	Scots Acres.	Engl. Acres.	Scots Acres.
A.	A. R. P.		A. R. P.
1	0..3.. 7.4[illegible]	51	40..2..11.02
2	1..3..14.8[illegible]	52	41..1..18.30
3	2..3..21.8[illegible]	53	42..0..25.57
4	3..3..28.10	54	42..3..32.85
5	4..3..35.37	55	43..3.. 0.12
6	5..3..42.65	56	44..2.. 7.40
7	6..3..10.92	57	45..1..14.67
8	8..1..18.20	58	46..0..21.95
9	7..3..25.47	59	46..3..29.22
10	7..3..32.75	60	47..2..36.50
11	8..3.. 0.02	61	48..2.. 3.77
12	8..3.. 7.30	62	49..1..11.05
13	10..1..14.57	63	50..0..18.32
14	11..0..21.85	64	50..3..25.60
15	11..3..29.12	65	51..2..32.87
16	12..3..36.40	66	52..2.. 0.15
17	13..3.. 3.67	67	53..1.. 7.42
18	14..3..10.95	68	54..0..14.70
19	15..2..18.22	69	54..3..21.97
20	16..3..25.50	70	55..2..29.25
21	16..2..32.77	71	56..1..36.52
22	17..2.. 0.05	72	57..1.. 3.80
23	18..1.. 7.32	73	58..0..11.07
24	19..0..14.60	74	58..3..18.35
25	19..3..21.87	75	59..2..25.62
26	20..2..29.15	76	60..1..32.90
27	21..1..36.42	77	61..1.. 0.17
28	22..1.. 3.70	78	62..0.. 7.45
29	23..0..10.97	79	62..3..14.72
30	23..3..18.25	80	63..2..22.00
31	24..2..25.52	81	64..1..29.27
32	25..1..32 80	82	65..0..36.55
33	26..1.. 0.07	83	66..0.. 3.82
34	27..0.. 7.35	84	66..3..11.10
35	27..3..14.62	85	67..2..18.37
36	28..2..21.90	86	68. 1..25.65
37	29..1..29,17	87	69..0..32.92
38	30..0..36,45	88	70..0.. 0.20
39	31..0.. 3.72	89	70..3.. 7.47
40	31..3..11.00	90	71..2..14.75
41	32..2..18.27	91	72..1..22.02
42	33..1..25.55	92	73..0..29.30
43	34..0..32.82	93	73..3..36.57
44	35..0.. 0.10	94	74..3.. 3.85
45	35..3.. 7.37	95	75..2..11.12
46	36..2..14.65	96	76..1..18.40
47	37..1..21.92	97	77..0..25.67
48	38..0..29.20	98	77..3..32.95
49	38..3..36.47	99	78..3.. 0.22
50	39..3.. 3.75	100	79..2.. 7.50

TABLE

Of Feet, answering to any number of Links in the Eng[illegible] 66 Feet.

Lks.	Ft.	In.	Lks.	Ft.	In.
1	0..	7.92	51	[illegible]	
2	1..	3.84	52	[illegible]	
3	1..11.76		53	[illegible]	
4	2..	7.68	54	[illegible]	
5	3..	3.60	55	36..	[illegible]
6	3..11.52		56	36..11.	[illegible]
7	4..	7.44	57	37..	2.16
8	5..	3.36	58	38..	[illegible]
9	5..11.28		59	38..11.	[illegible]
10	6..	7.20	60	39..	7.[illegible]
11	7..	3.12	61	40..	3.[illegible]
12	7..11.04		62	40..11.	[illegible]
13	8..	6.96	63	41..	6.[illegible]
14	9..	2.88	64	42..	2.[illegible]
15	9..10.80		65	42..10.	[illegible]
16	10..	6.72	66	43..	6.72
17	11..	2.64	67	44..	2.64
18	11..10.56		68	44..10.	[illegible]
19	12..	6.48	69	45..	6.[illegible]
20	13..	2.40	70	46..	2.[illegible]
21	13..10.32		71	46..10.	[illegible]
22	14..	6.24	72	47..	6.[illegible]
23	15..	2.16	73	48..	2.16
24	15..10.08		74	48..10.	[illegible]
25	16..	6.00	75	49..	6.00
26	17..	1.92	76	50..	1.92
27	17..	9.84	77	50..	9.84
28	18..	5.76	78	51..	5.76
29	19..	1.68	79	52..	1.68
30	19..	9.60	80	52..	9.60
31	20..	5.52	81	53..	5.52
32	21..	1.44	82	54..	1.44
33	21..	9.36	83	54..	9.36
34	22..	5.28	84	55..	5.28
35	23..	1.20	85	56..	1.20
36	23..	9.12	86	56..	9.12
37	24..	5.04	87	57..	5.04
38	25..	0.96	88	58..	0.96
39	25..	8.88	89	58..	8.88
40	26..	4.80	90	59..	4.80
41	27..	0.72	91	60..	0.72
42	27..	8.64	92	60..	8.64
43	28..	4.56	93	61.	4.56
44	29..	0.48	94	62..	0.48
45	29..	8.40	95	62..	8.40
46	30..	4.32	96	63..	4.32
47	31..	0.24	97	64..	0.24
48	31..	8.16	98	64..	8.16
49	32..	4.08	99	65..	4.08
50	33..	0.00	100	66..	0.00

TABLE

Of Feet, answering to any number of Links in the Scots Chain of 74 Feet.

Lks.	Ft.	In.	Lks.	Ft.	In.
1	0..	8.88	51	37..	8.88
2	1..	5.76	52	38..	5.76
3	2..	2.64	53	39..	2.64
4	2..11.52		54	39..11.	52
5	3..	8.40	55	40..	8.40
6	4..	5.28	56	41..	5.28
7	5..	2.16	57	42..	2.16
8	5..11.04		58	42..11.	04
9	6..	7.92	59	43..	7.92
10	7..	4.80	60	44..	4.80
11	8..	1.68	61	45..	1.68
12	8..10.56		62	45..10.	56
13	9..	7.44	63	46..	7.44
14	10..	4.32	64	47..	4.32
15	11..	1.20	65	48..	1.20
16	11..10.08		66	48..10.	08
17	12..	6.96	67	49..	6.96
18	13..	3.84	68	50..	3.84
19	14..	0.72	69	51..	0.72
20	14..	9.60	70	51..	9.60
21	15..	6.48	71	52..	6.48
22	16..	3.36	72	53..	3.36
23	17..	0.24	73	54..	0.24
24	17..	9.12	74	54..	9.12
25	18..	6.00	75	55..	6.00
26	19..	2.88	76	56..	2.88
27	19..11.76		77	56..11.	76
28	20..	8.64	78	57..	8.64
29	21..	5.52	79	58..	5.52
30	22..	2.40	80	59.	2.40
31	22..11.28		81	59..11.	28
32	23..	8.16	82	60..	8.16
33	24..	5.04	83	61..	5.04
34	25..	1.92	84	62..	1.92
35	25..10.80		85	62..10.	80
36	26..	7.68	86	63..	7.68
37	27..	4.56	87	64..	4.56
38	28..	1.44	88	65..	1.44
39	28..10.32		89	65..10.	32
40	29..	7.20	90	66..	7.20
41	30..	4.08	91	67..	4.08
42	31..	0.96	92	68.	0.96
43	31..	9.84	93	68..	9.84
44	32..	6.72	94	69.	6.72
45	33..	3.60	95	70..	3.60
46	34..	0.48	96	71.	0.48
47	34..	9.36	97	71..	9.36
48	35..	6.24	98	72..	6.24
49	36..	3.12	99	73..	3.12
50	37..	0.00	100	74..	0.00

TABLE

Of Feet, answering to any number of Links in the Scotch Chain of 74.4 Feet.

Lks.	Ft.	In.	Lks.	Ft.	In.
1	0..	8 92	51	37..	11.36
2	1..	5.85	52	38..	8.28
3	2..	2.78	53	39..	5.19
4	2..	11.71	54	40..	2.11
5	3..	8.64	55	40..	11.04
6	4..	5.56	56	41..	7.96
7	5..	2.48	57	42..	4.89
8	5..	11.42	58	43..	1.82
9	6.	8.35	59	43..	10.73
10	7..	5.28	60	44..	7.66
11	8..	2.20	61	45.	4.60
12	8..	11.13	62	46..	1.53
13	9..	8.06	63	46..	10.46
14	10..	4.99	64	47..	7.38
15	11..	1.92	65	48..	4.32
16	11..	10.84	66	49..	1.24
17	12..	7.77	67	49..	10.17
18	13..	4.70	68	50..	7.10
19	14..	1.63	69	51..	4.03
20	14..	10.56	70	52..	0.96
21	15..	7.48	71	52..	9.88
22	16..	4.41	72	53..	6.81
23	17..	1.34	73	54..	3.74
24	17..	10.27	74	55..	0.67
25	18..	7.20	75	55..	9.60
26	19..	4.12	76	56..	6.52
27	21..	1.05	77	57..	3.45
28	20..	9.98	78	58..	0.38
29	21..	6.91	79	58..	9.31
30	22..	3.84	80	59..	6.24
31	23..	0.76	81	60..	3.16
32	23..	9.69	82	61..	0.09
33	24..	6.62	83	61..	9.02
34	25..	3.55	84	62..	5.95
35	26..	0.48	85	63..	2.88
36	26..	9.40	86	63..	11.80
37	27..	6.33	87	64..	8.73
38	28..	3.26	88	65..	5.66
39	29..	0.19	89	66.	2.59
40	29..	9.12	90	66..	11.52
41	30..	6.04	91	67..	8.44
42	31..	2.97	92	68..	5.37
43	31..	11.90	93	69..	2.30
44	32..	8.83	94	69..	11.23
45	33..	5.76	95	70..	8.16
46	34..	2.68	96	71.	5.08
47	34..	11.61	97	72..	2.01
48	35..	8.54	98	72..	10.91
49	36..	5.47	99	73..	7.87
50	37..	2.40	100	74..	4.80

TABLE

...wering to any number ...in the Irish Chain of

Lks.	Ft.	In	Lks.	Ft.	In.
1	0..	10.08	51	42..	10.08
2	1..	8.16	52	43..	8.16
3	2..	6.24	53	44..	6.24
4	3..	4.32	54	45..	4.32
5	4..	2.40	55	46..	2.40
6	5..	0.48	56	47..	0.48
7	5..	10.56	57	47..	10.56
8	6..	8.64	58	48..	8.64
9	7..	6.72	59	49..	6.72
10	8..	4.80	60	50..	4.80
11	9..	2.88	61	51..	2.88
12	10..	0.96	62	52..	0.96
13	10..	11.04	63	52..	11.04
14	11..	9.12	64	53..	9.12
15	12..	7.20	65	54..	7.20
16	13..	5.28	66	55..	5.28
17	14..	3.36	67	56..	3.36
18	15..	1.44	68	57..	1.44
19	15..	11.52	69	57..	11.52
20	16..	9.60	70	58..	9.60
21	17..	7.68	71	59..	7.68
22	18..	5.76	72	60..	5.76
23	19..	3.84	73	61..	3.84
24	20..	1.92	74	62..	1.92
25	21..	0.00	75	62..	0.00
26	21..	10.08	76	63..	10.08
27	22..	8.16	77	64..	8.16
28	23..	6.24	78	65..	6.24
29	24..	4.32	79	66..	4.32
30	25..	2.40	80	67..	2.40
31	26..	0.48	81	68..	0.48
32	26..	10.56	82	68..	10.56
33	27..	8.64	83	69..	8.64
34	28..	6.72	84	70..	6.72
35	29..	4.80	85	71..	4.80
36	30..	2.88	86	72..	2.88
37	31..	0.96	87	73..	0.96
38	31..	11.04	88	73..	11.04
39	32..	9.12	89	74..	9.12
40	33..	7.20	90	75..	7.20
41	34..	5.28	91	76..	5.28
42	35..	3.36	92	77..	3.36
43	36..	1.44	93	78..	1.44
44	36..	11.52	94	78..	11.52
45	37..	9.60	95	79..	9.60
46	38..	7.68	96	80..	7.68
47	39.	5.76	97	81..	5.76
48	40..	3.84	98	82..	3.84
49	41..	1.92	99	83..	1.92
50	42..	0.00	100	84..	0.00

54.—It will now appear that for taking the measurements of any considerable extent of land it is more convenient to make only those necessary for constructing a plan, and afterwards by the methods given in Art. 52, find the respective areas of the fields or plots as marked out either by the boundaries of the land or its fences. But more particularly to illustrate the advantages of this mode of finding the areas, we shall add the following actual survey of a Villa and Farm, the plan of which was wholly constructed from chain measurements and offsets only.

As upon the figure, the primary triangles measured were, A B C, C B D, C D E, E D F, E F G, F G H, D L I, L I K, and the quadrilateral F I K H. The point E was upon the straight line C G, by which the three triangles C D E, E D F, E F G, were comprised in the quadrilateral C D F G. The secondary lines besides the primary triangles for completing the plan, were *a* F, *b c*, *d e*, *f g*, and *h g*, all within triangle D E F, with *k* within C B D.

From the disposition of the primary triangles, the sides of these afford sufficient data with the necessary offsets to the boundary, for calculating without a plan the whole area of the lands: but this with the subsidiary measurements, are not enough for calculating the respective areas of all the fields, without the addition of a number of others, or Scale measurements. Hence it is manifest that for constructing a plan only of many fields, the measuring may be much less than if to afford data for calculating the area of each field respectively: for by the shortest method possible, had the different plots of this plan been measured so as to have given data for calculating each field, this would have required to have measured every one after the manner of Art. 46. Ex. 2 and 3; or at least by Art. 48, Ex. 7 and 8.

As upon the plan every survey made of many fields together, should afford data in the actual measurements for calculating the whole area from the series of triangles covering the lands by which a correction is afforded for the scale measurements of each field; for the collective contents of the last should always agree very nearly with the first, and when this is obtained, after having the accuracy of the actual measurements proven by proof lines,* there can be no doubt that the results of all the parts calculated by the scale are true.

If we now take a view of the different methods of surveying with the chain only, which I have heretofore exposed, it is very evident for to obtain a complete verification of all the operations with the least labour, that a delineation of the lands is always indispensable; that is to say, when the extent is beyond the simple cases, (as art. 48.) for we can construct the plan with a great deal less measurement than that which is requisite to find the [illegible] while this at the same time will [illegible] total area, which must be always true after the measurements are verified upon the plan by proof lines; also the scale measurements of the plan, with the calculations therefrom, may be always the simplest, and a complete verification obtained by their collective contents agreeing with that of the primary triangles; whereas, were each field measured so as to afford data for calculating the areas severally, this would require in the event of making a calculation by one diagonal, another at least with the proof lines to verify it, so that this method under every circumstance would cause many more actual measurements than the above, and two calculations from different data before the results could be considered true.

* The proof lines are not shown upon the figure, but only those necessary for constructing the plan.

What is above advanced, shows clearly the bad practice of the method of measuring with the chain and cross staff as exemplified in Art. 47, and in the works referred to therein, for in this method, proof lines cannot be made so as to afford data for two different calculations, but its accuracy depending wholly upon the first measurement alone, and without any verification whatever upon this; therefore whatever errors or omissions have been committed in the measurements, these cannot be corrected.

ERRATA.

Page 5, *line third from the top, for* D F F, *read* D, E, F.
7, *line third from the bottom, for* D Γ, *read* D F
13, *line fourth from the top, for* bisect *read* trisect.
14, *line second from the top for* A B C, *read* A, B, C.
16, *line third from the top, for* A O, indefinitely cutting, *read* A O indefinitely, cutting.
Ib. line tenth from the top, for circumference, *read* circumferences
20, *line eighth from the top, for* f, g: *read* f ".
22 *line ninth from the top, for* b, c, a, e, f, g: *read* b, c, d, e, f, g.
Ib. line fourteenth from the top, for A B, *read* A D
27, *line eighth from the bottom, for* and is parallel, *read* and F K is parallel.
32, *line seventh from the top, for* A I F, *read* A I.
Ib. line seventeenth from the top, for K E N, *read* F N K.
Ib line twentieth from the top, for A L M B and the, *read* A L M B with the
Ib. line twenty-seventh from the top, [illegible] L M, [illegible] C L M.
50, *line at bottom, for* 42547 *read* [illegible]
Ib line at bottom, for 1 9 2 1, *read* 19621.
55, *line nineteenth from bottom, for* 11930, *read* 10830.
66, *line twelfth from bottom, for* 433, *read* 533
68, *line ninth and tenth from bottom, for* [illegible]
79, *line eleventh from the top, for* 8.88, *read* [illegible]
8 , *line at bottom, for* $\frac{\text{D I}}{2}$ *read* D P.
86 *line sixth from top, for* I P *read* D b
90 Art 9 *for t line, for* I K *read* F
96, Art 96 *fifth line, for* I D F. *read* F D E.
98, *line third from the bottom, for* 95 nd 4 , *read* 135 and 142.
100 *line eleventh from the top, for* A C *read* A c
105, *line fourth from the bottom, for* a D B. *read* a t B.
118 *line third from the top, for* 575, *read* 7 .
1 8, *line ninth from the bottom, for* 280 *read* 330.
1 9 *line s nth from the top, for* 7, *read* 6.
Ib. line sixth from the bottom, for 4816, *read* 6204.
Ib line fourth from the bottom, for 18278, *read* 19666.
Ib. line second from the bottom, for 91 9, *read* 9843.
Ib. bottom line, for 1 30501=1..1 88, *read* 1 31198=1. 1 9 9.
140, *line fifteenth from the top, for* e l k, *read* c l h
146, *line tenth from the bottom, for* 17 63, *read* 1786.3.
Ib. line second from the bottom, for $\frac{a \times b}{\quad}{}^{2}$ *read* $\left(\frac{a+b}{2}\right)^{2}$
150, *two lines at the top, read* intersection 4, through 4 and parallel to 5.3; and 5 through 5 parallel to 6.4; and 6 through 6 parallel to 6 5, and 7 through 7 parallel to 8.6; and lastly join 8.7.
157, *line sixth and seventh from the top, for* $\frac{20\,a\,b+10^{\ 2}}{20\,b-10\,b^{2}}$ *read* $\frac{20\,a\,b-10\,b^{2}}{20\,a\,b+10\,b^{2}}$
Ib. line fourteenth from the top, for subtracted, *read* added.
Ib. line fifth from the bottom, for added, *read* subtracted.

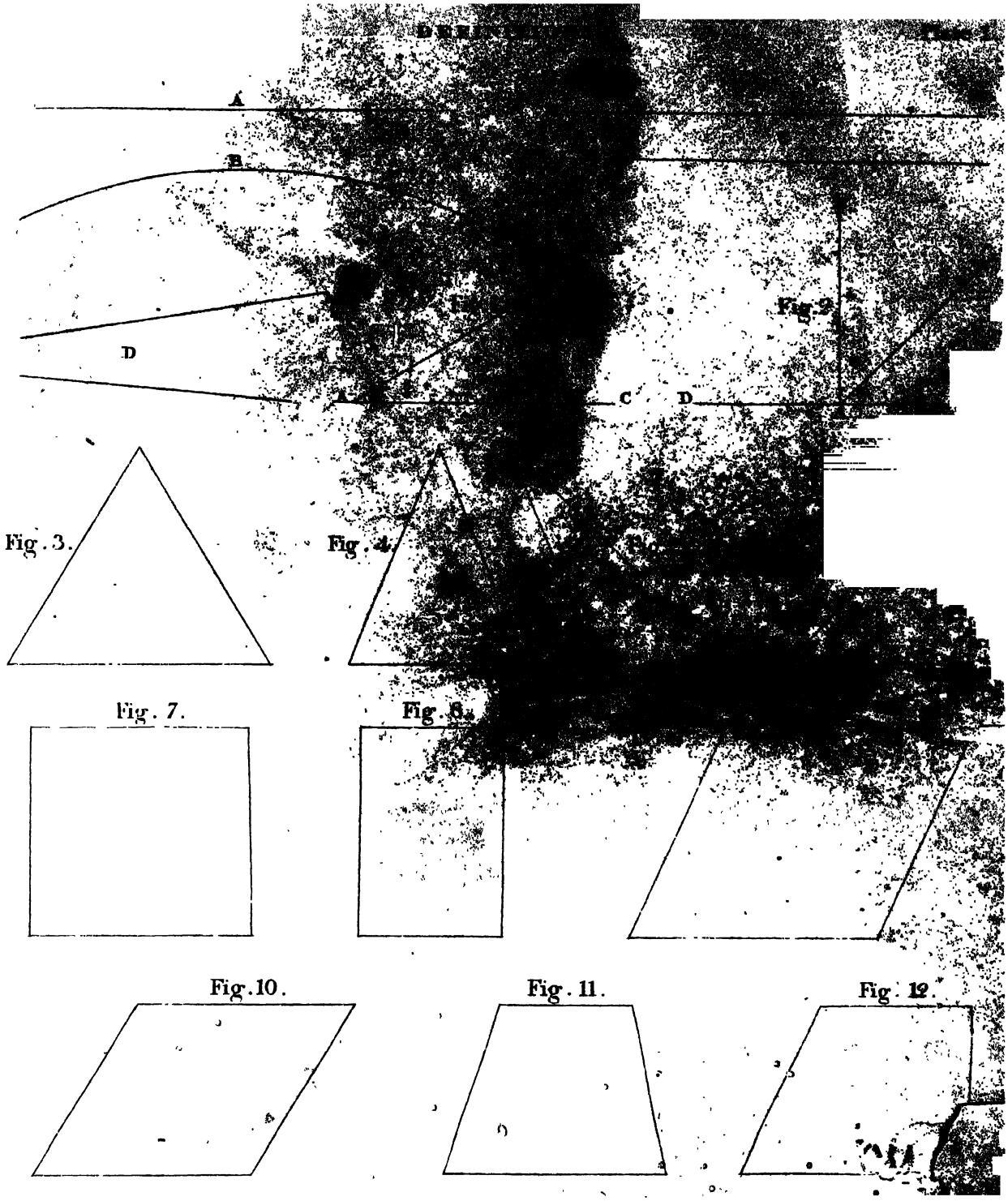

Fig. 3.

Fig. 4.

Fig. 7.

Fig. 8.

Fig. 10.

Fig. 11.

Fig. 12.

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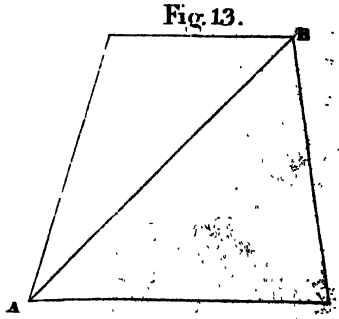
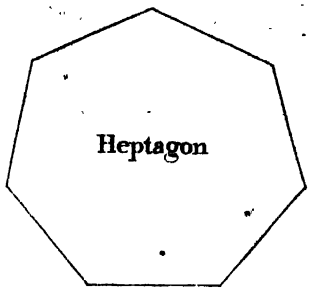
Fig. 13.





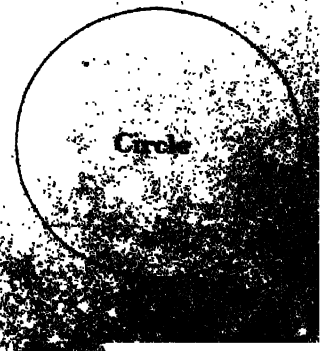
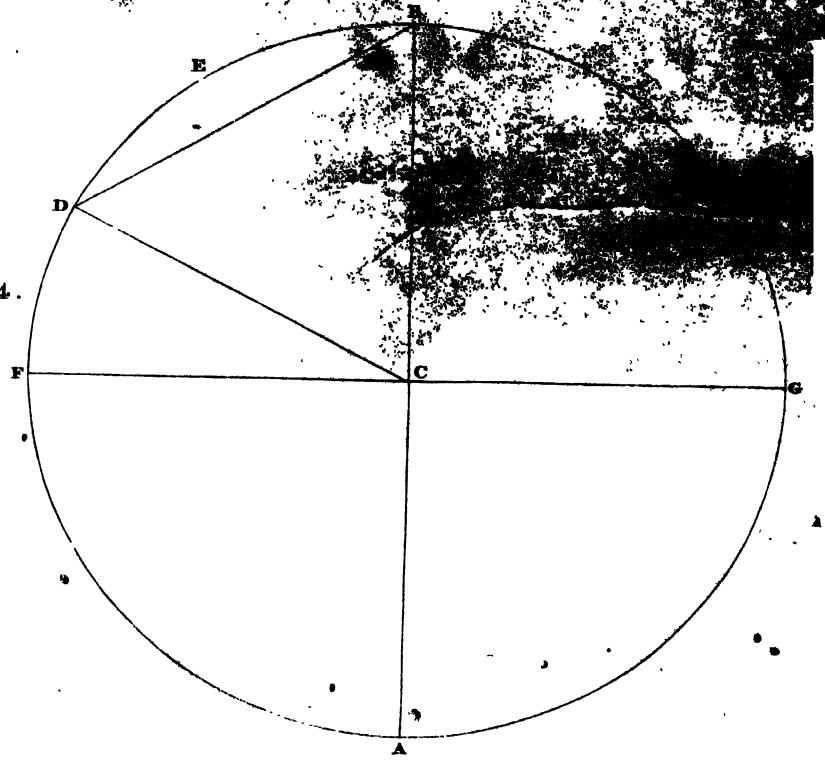


Fig. 14.



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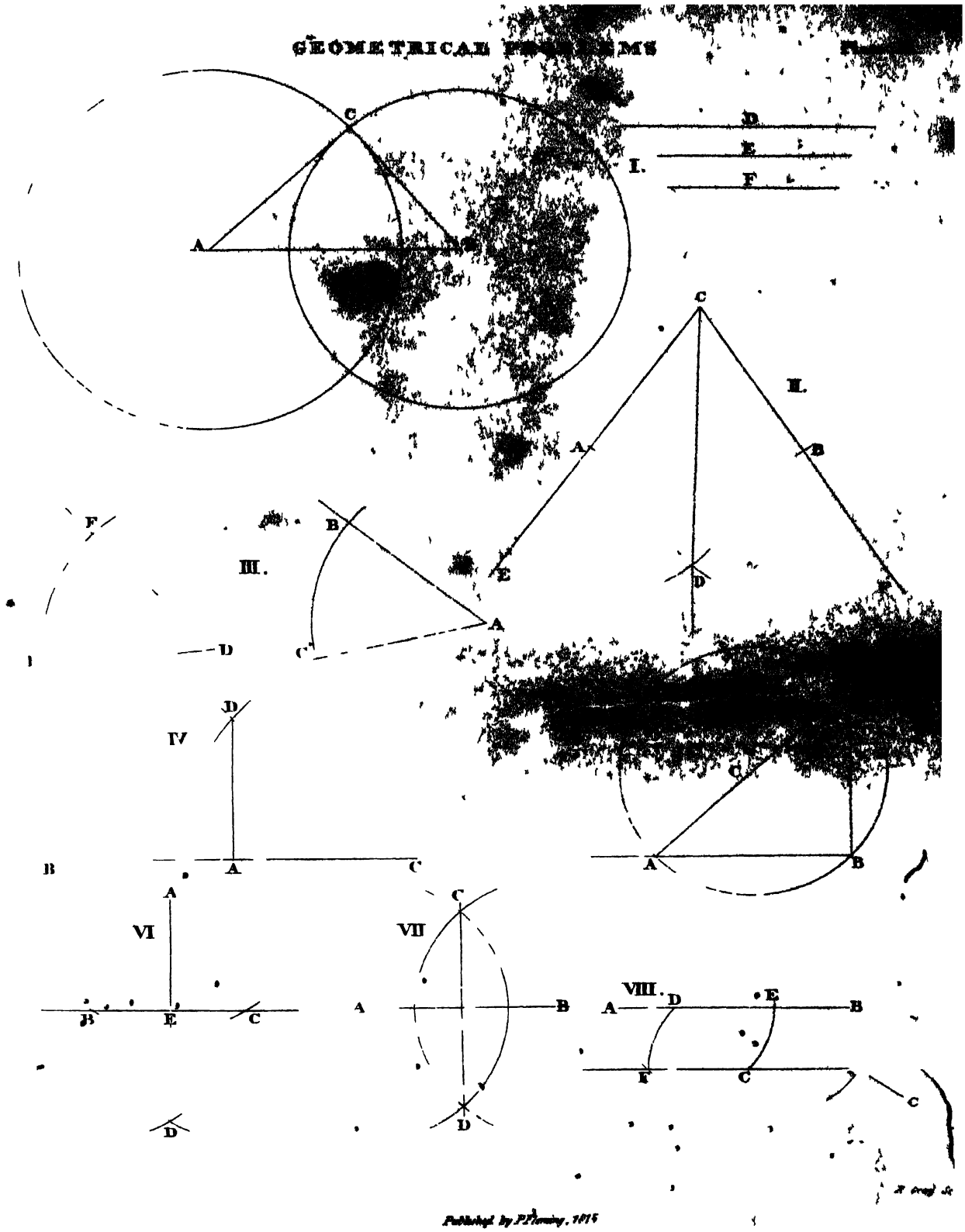

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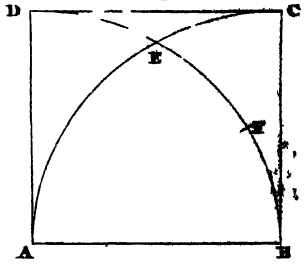

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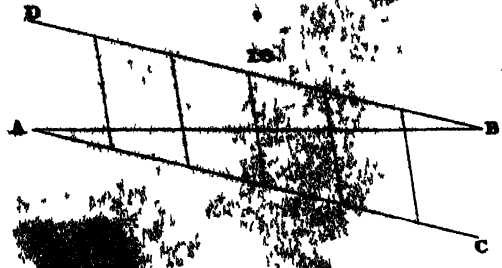

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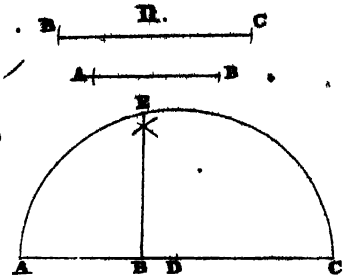

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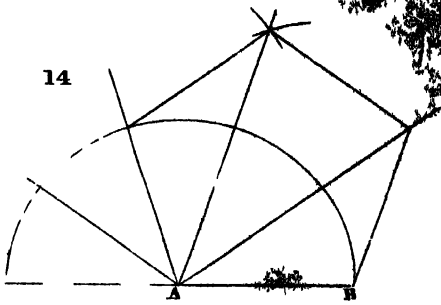

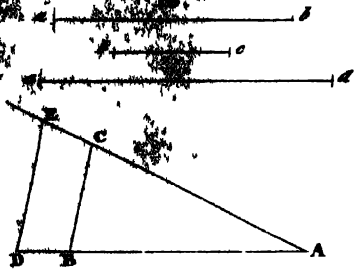

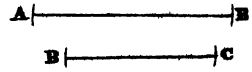

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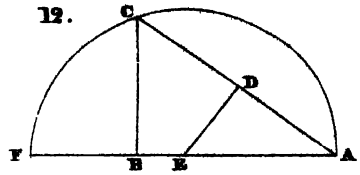

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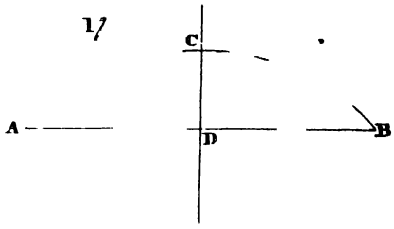

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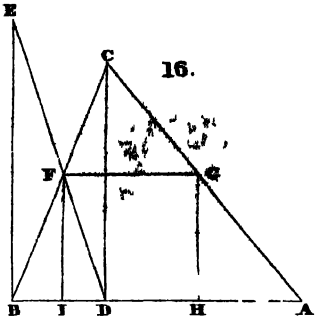

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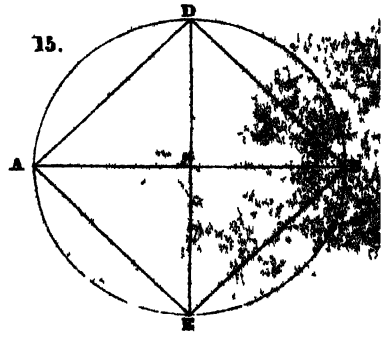

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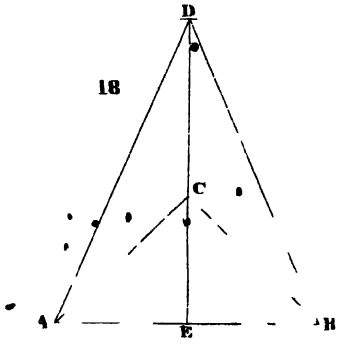

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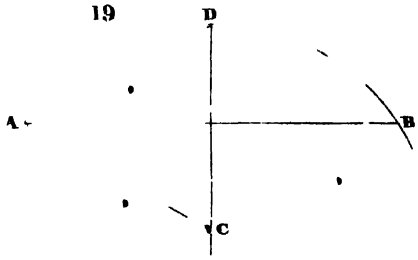

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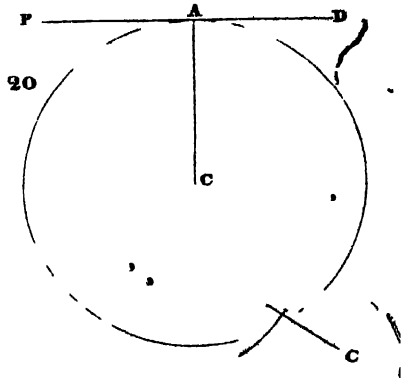

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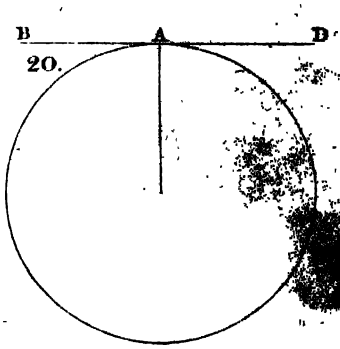

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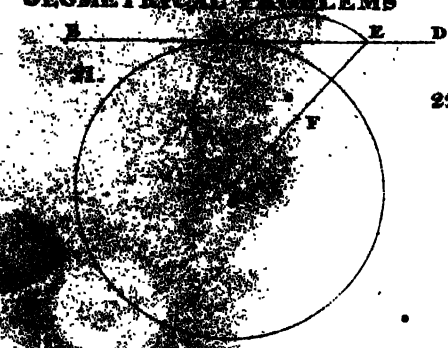

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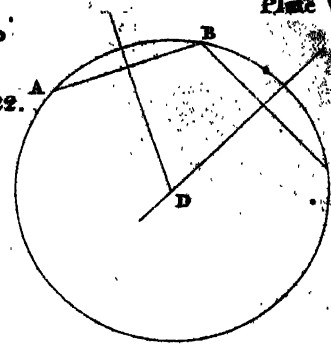

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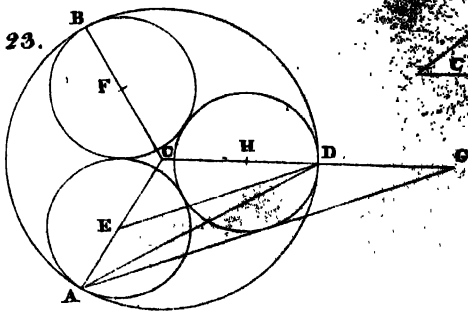

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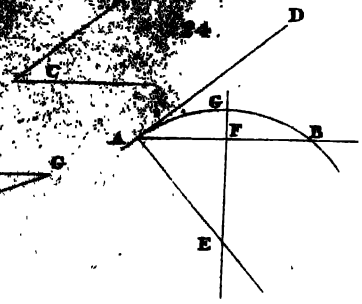

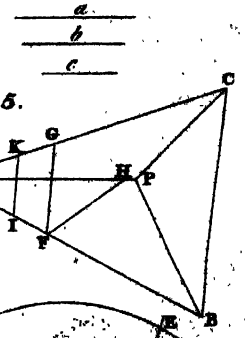

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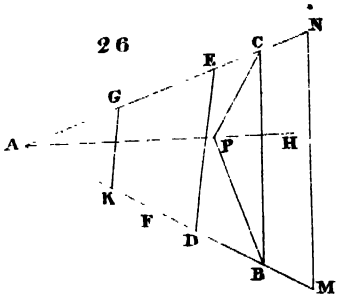

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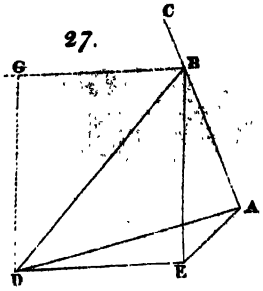

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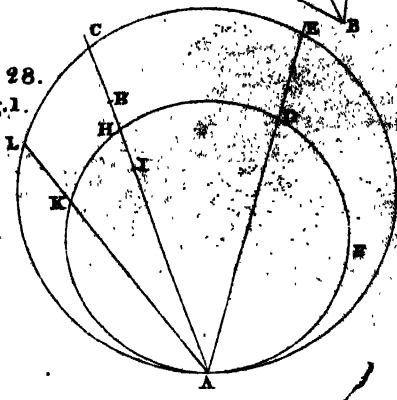

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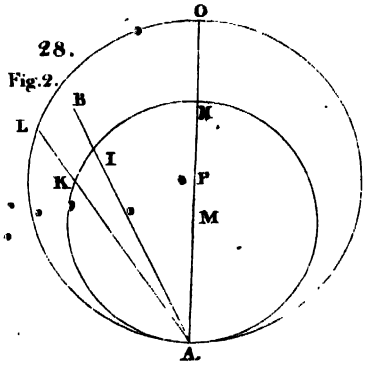

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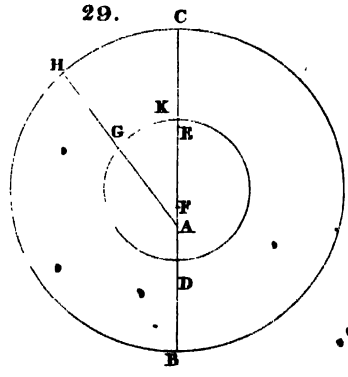

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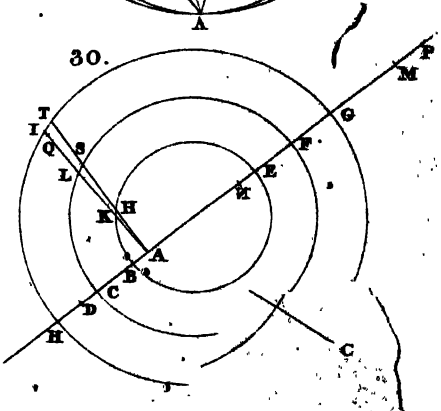

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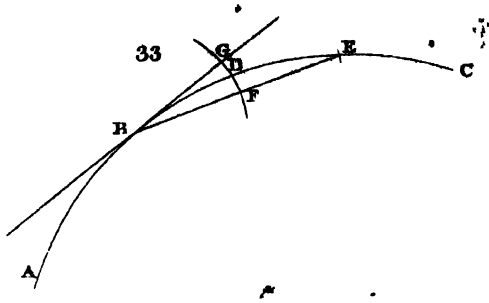

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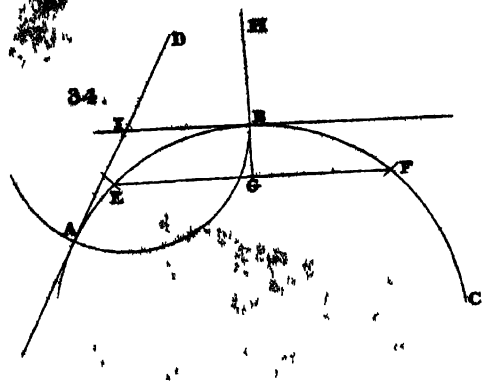

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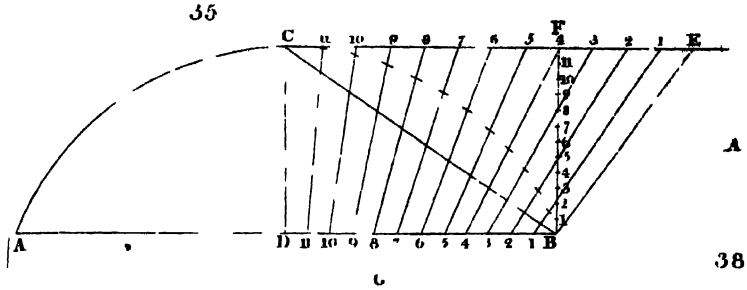

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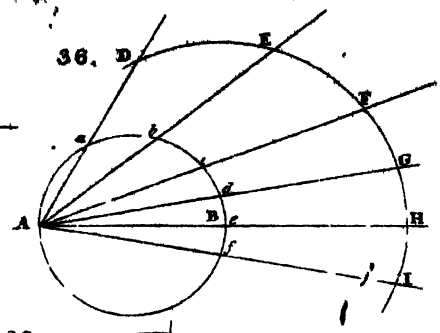

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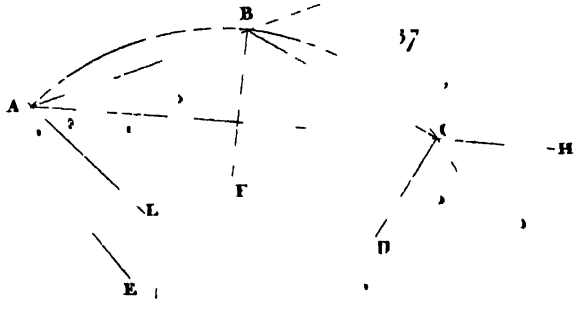

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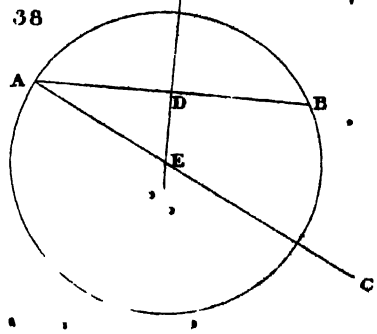

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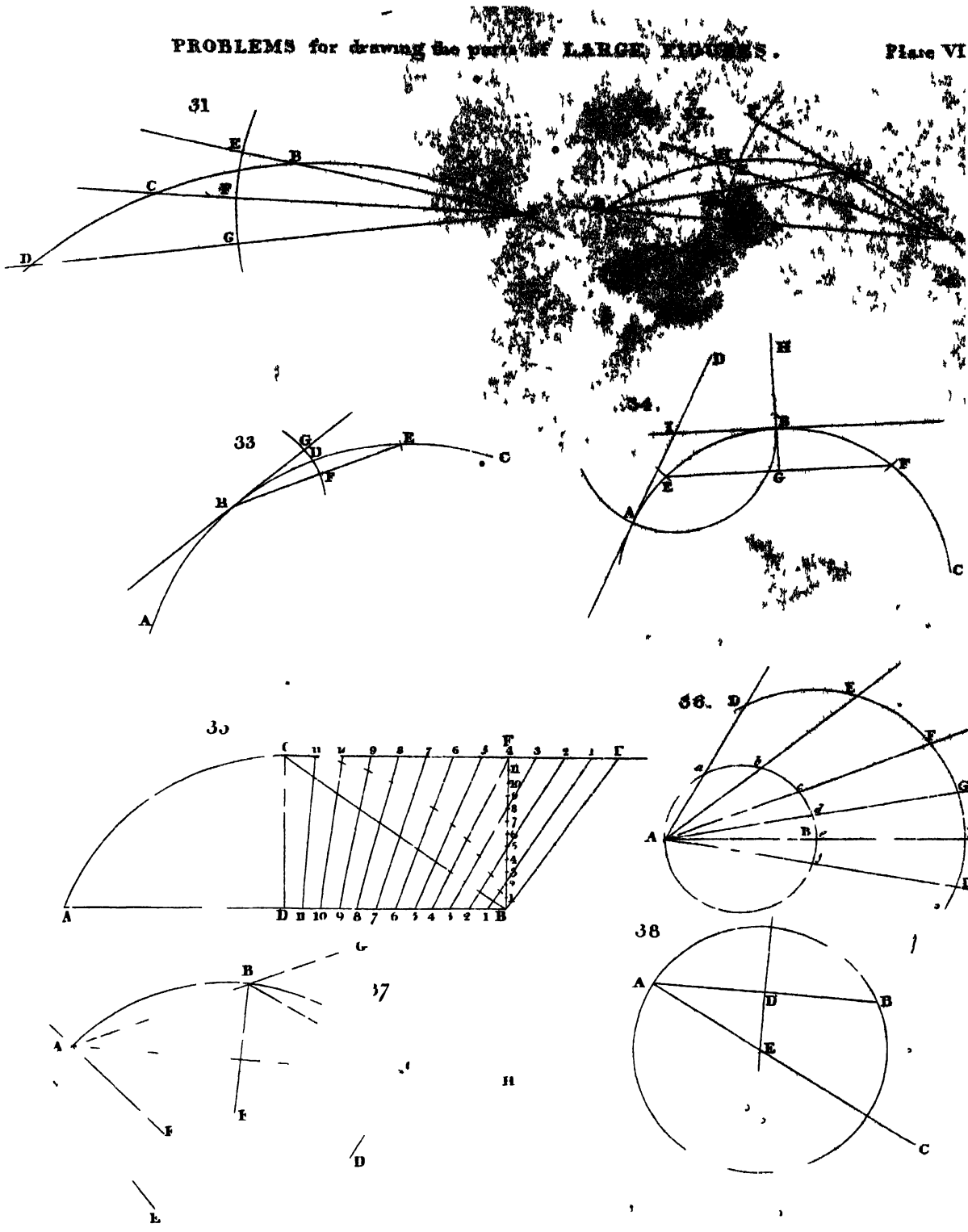

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PROBLEMS for drawing PLANE FIGURES.

39.

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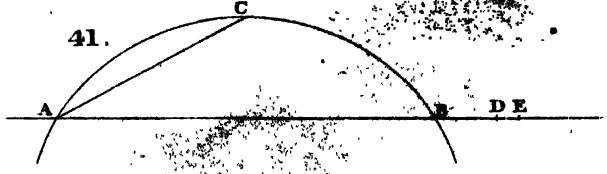

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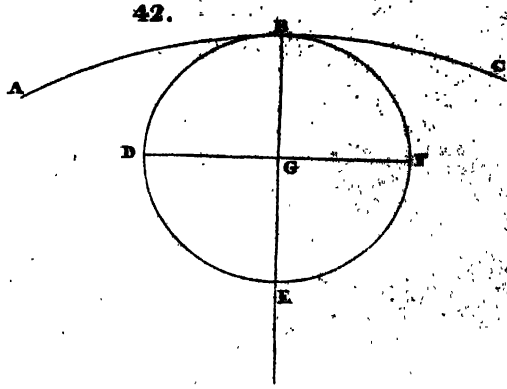

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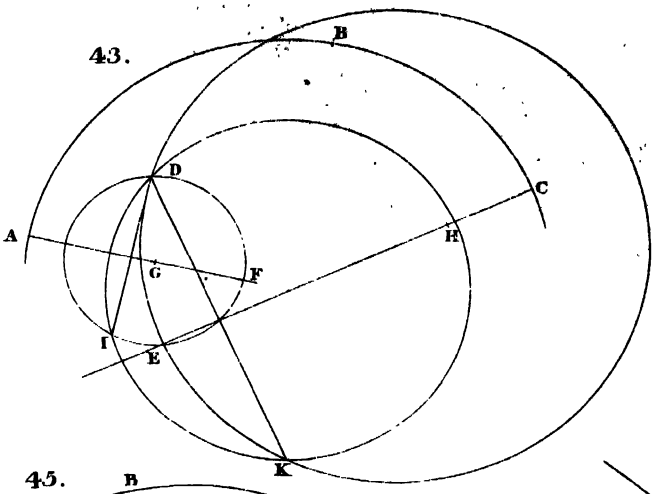

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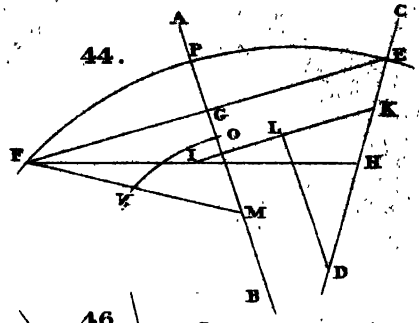

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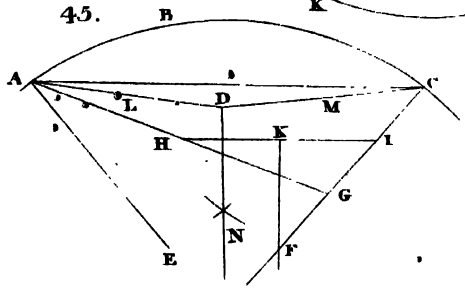

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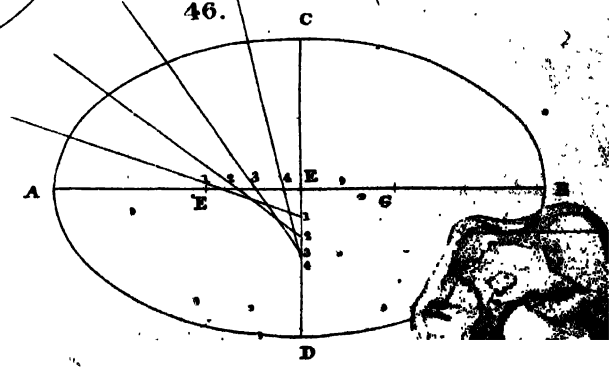

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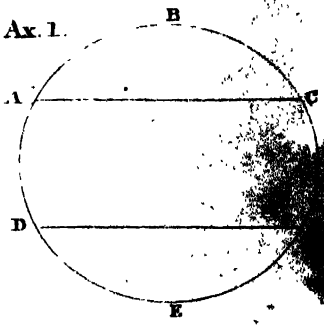

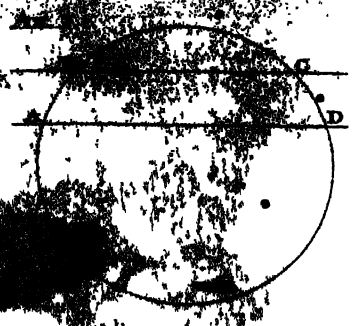

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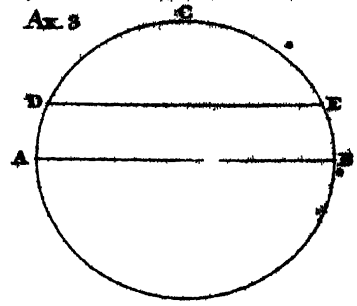

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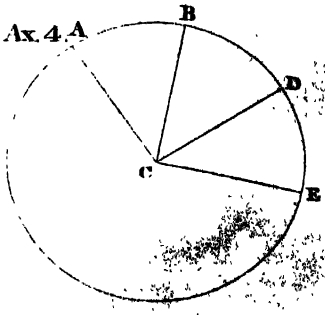

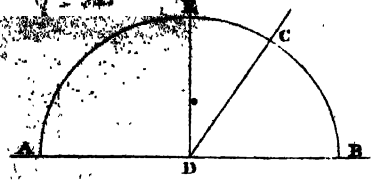

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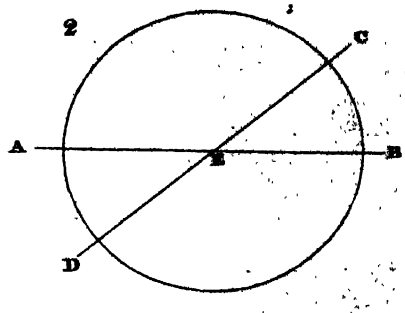

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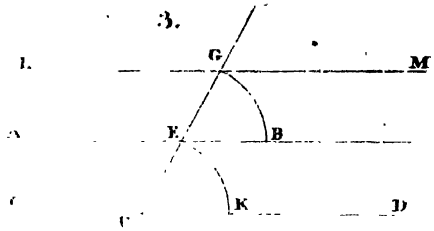

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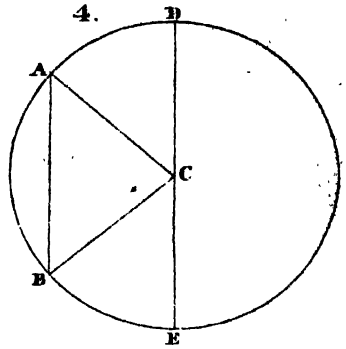

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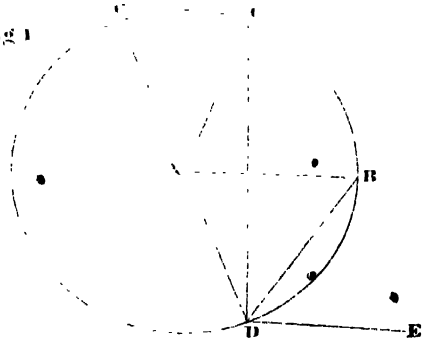

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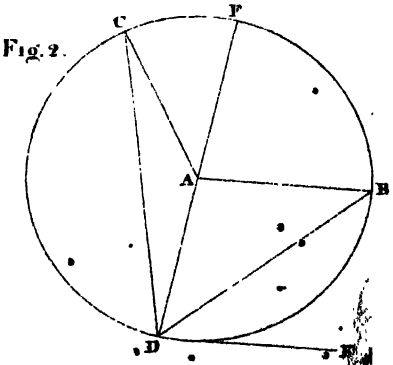

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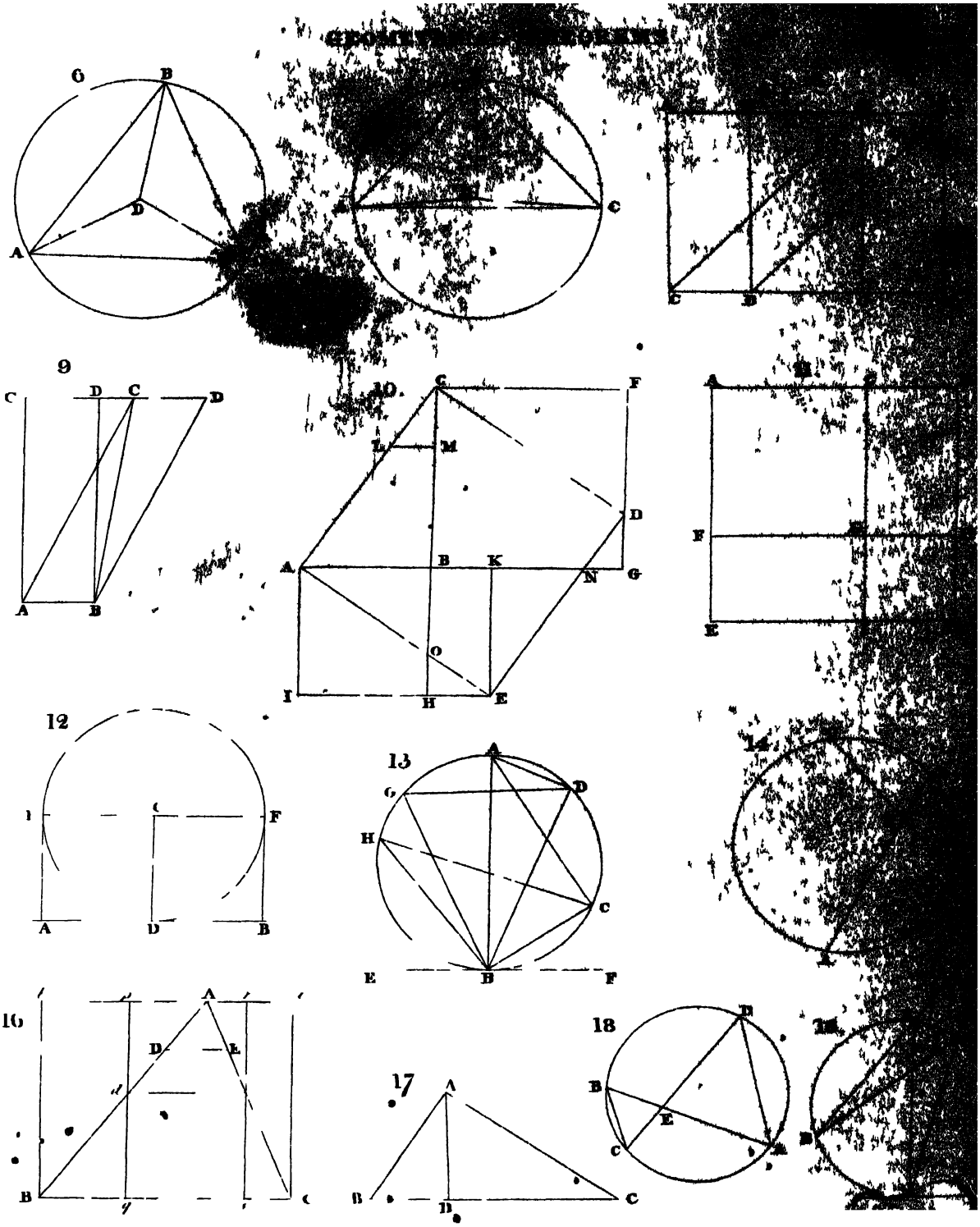

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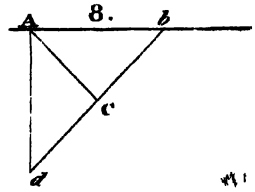
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A
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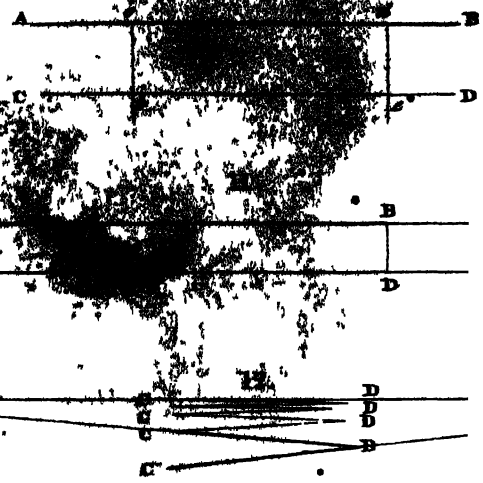
d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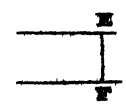
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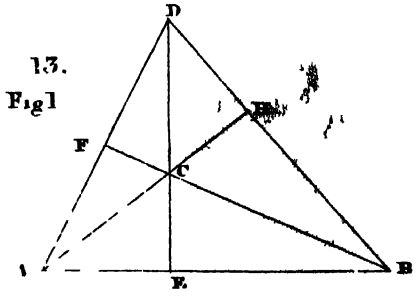
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Fig 1
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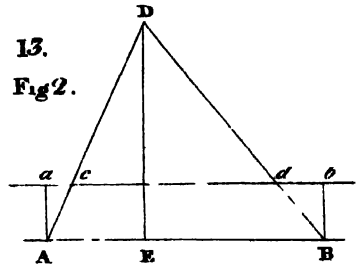
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Fig 2.
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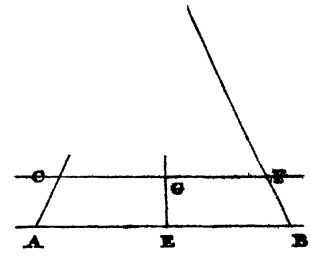
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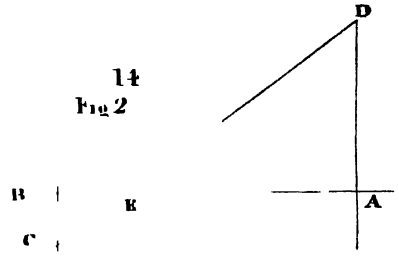
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Fig 2
D
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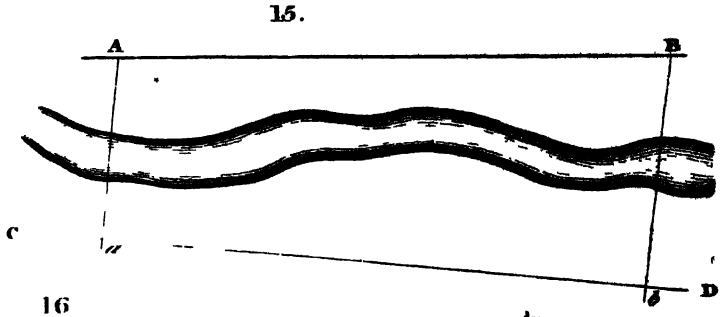
15.
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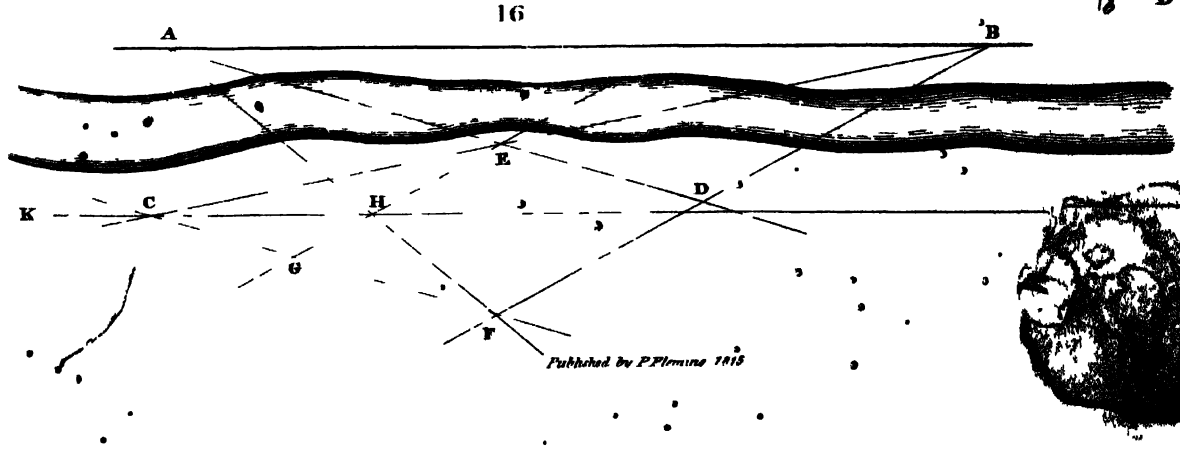
C
D


16
A
B
E
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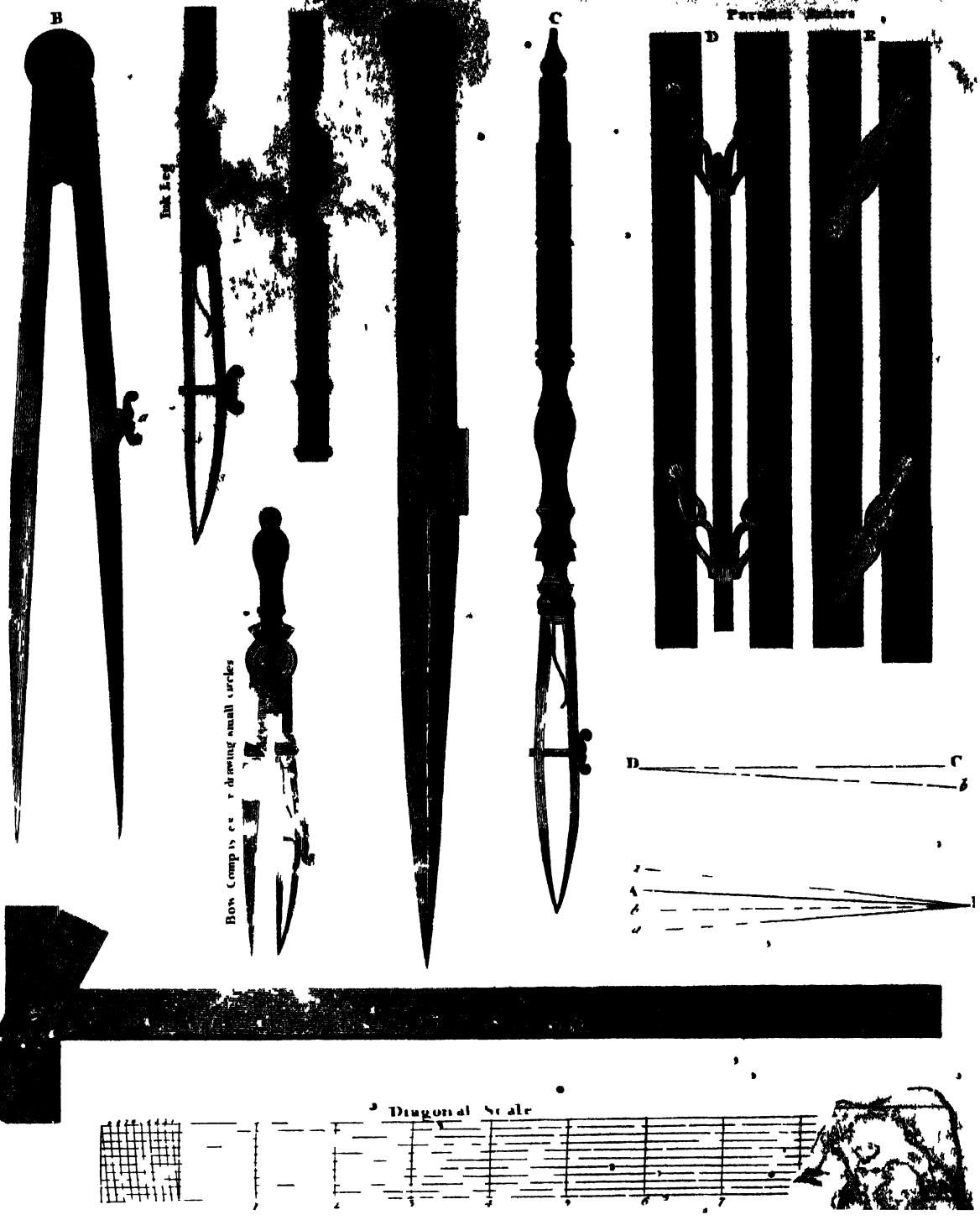

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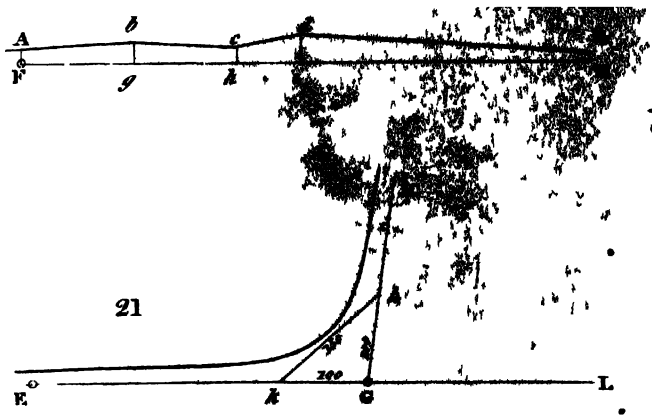

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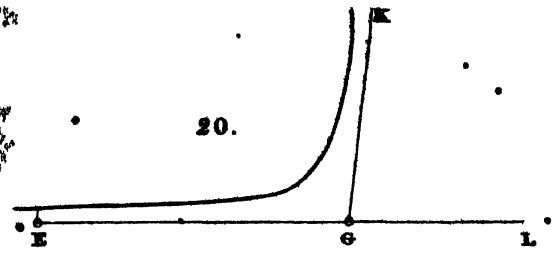

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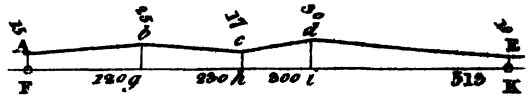
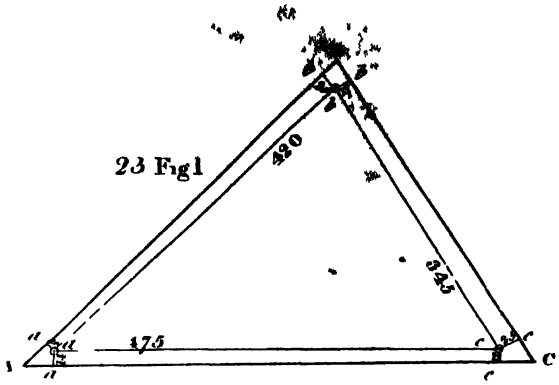
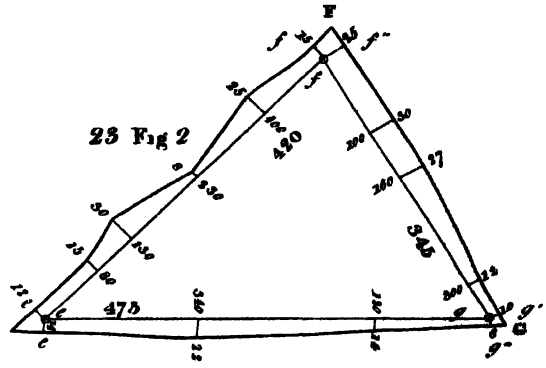


23 Fig 1



23 Fig 2



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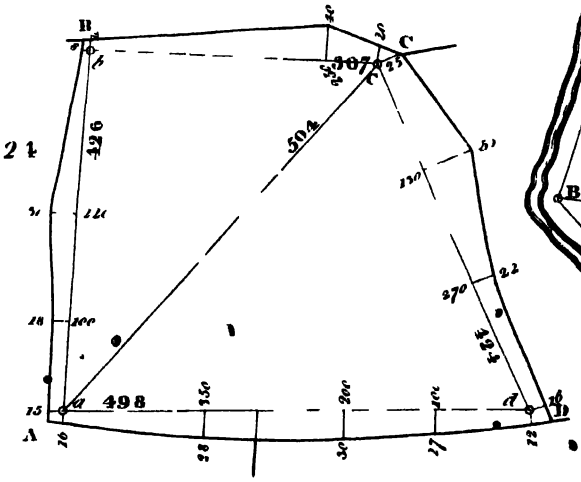

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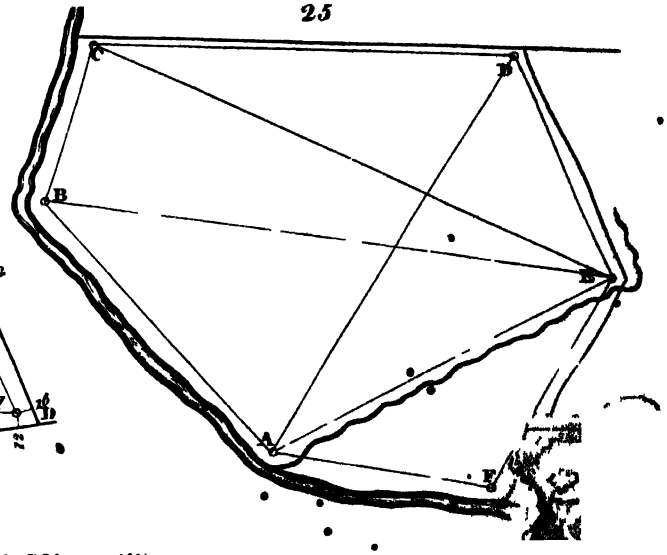

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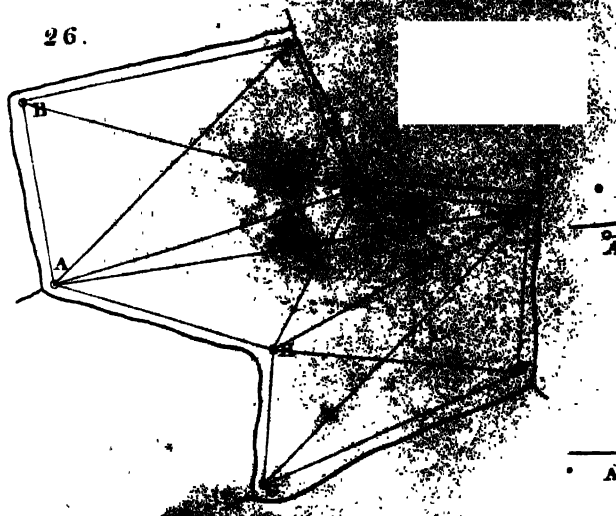
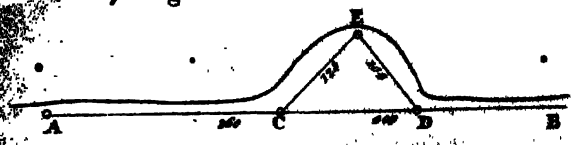
LAND SURVEYING, with the CHAIN only. Plate XII.

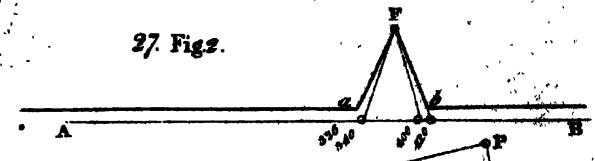
26.



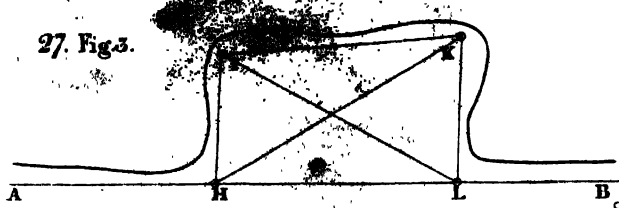
27. Fig.1.



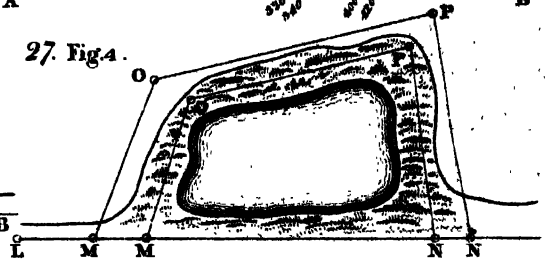
27. Fig.2.



27. Fig.3.



27. Fig.4.



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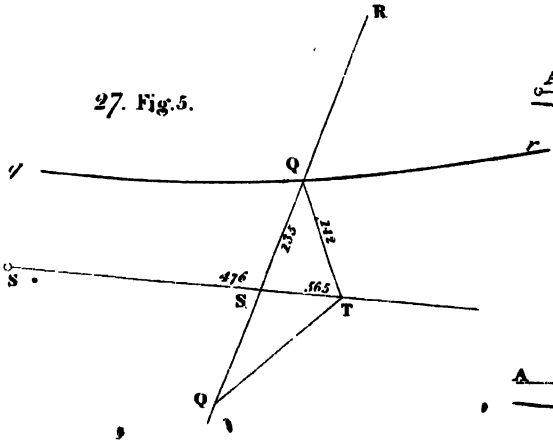
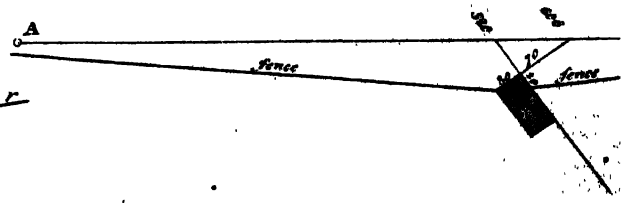
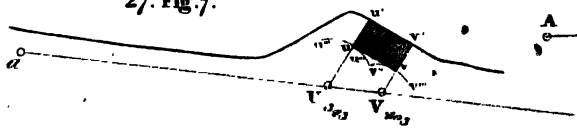


27. Fig.6.

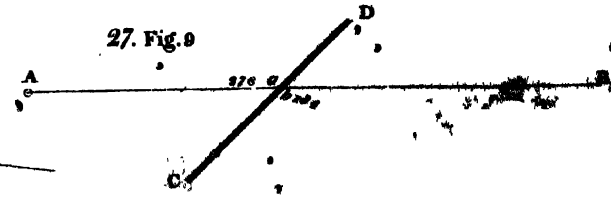


27. Fig.7.



27. Fig.8.

27. Fig.9



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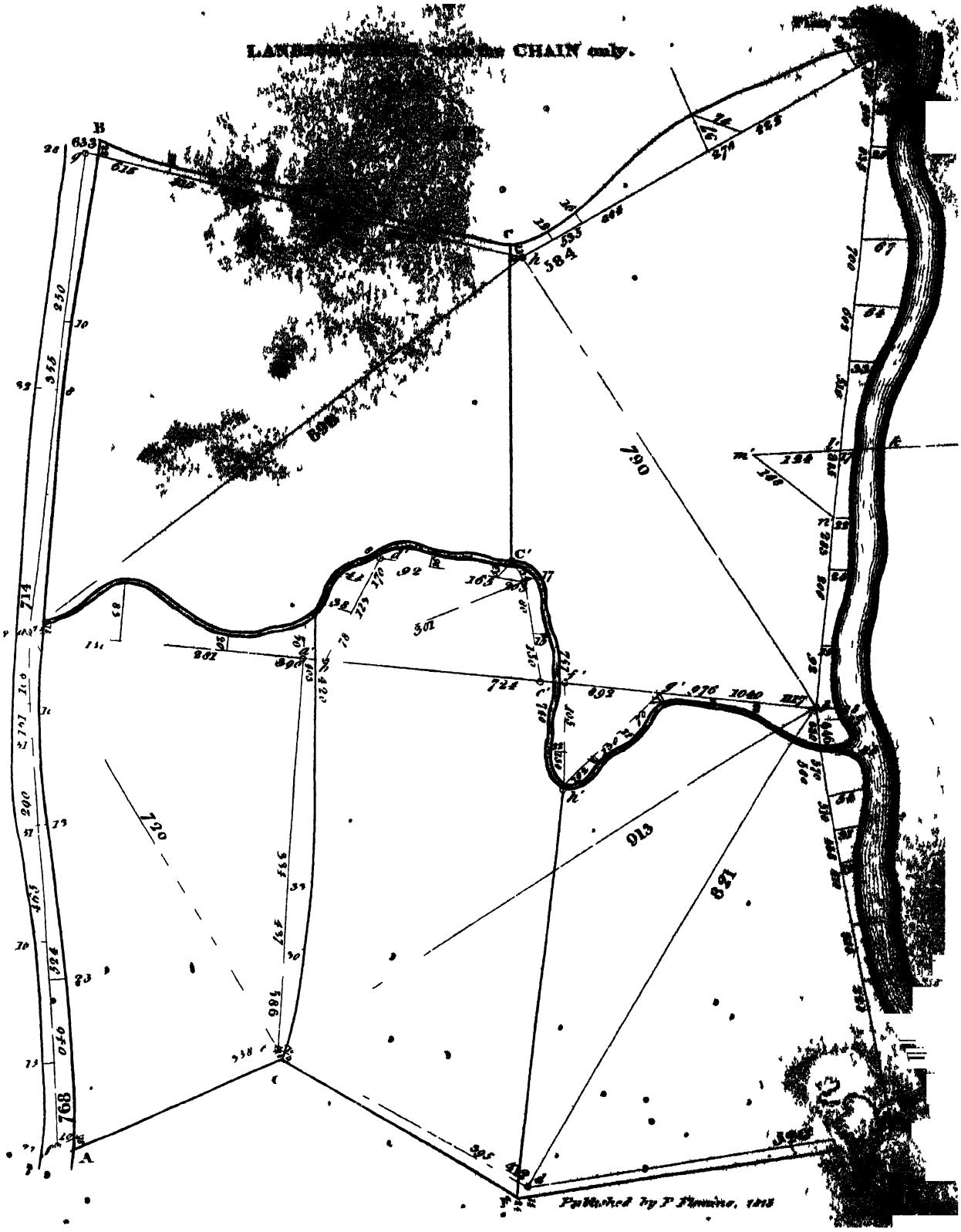
LANE
CHAIN only.
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720
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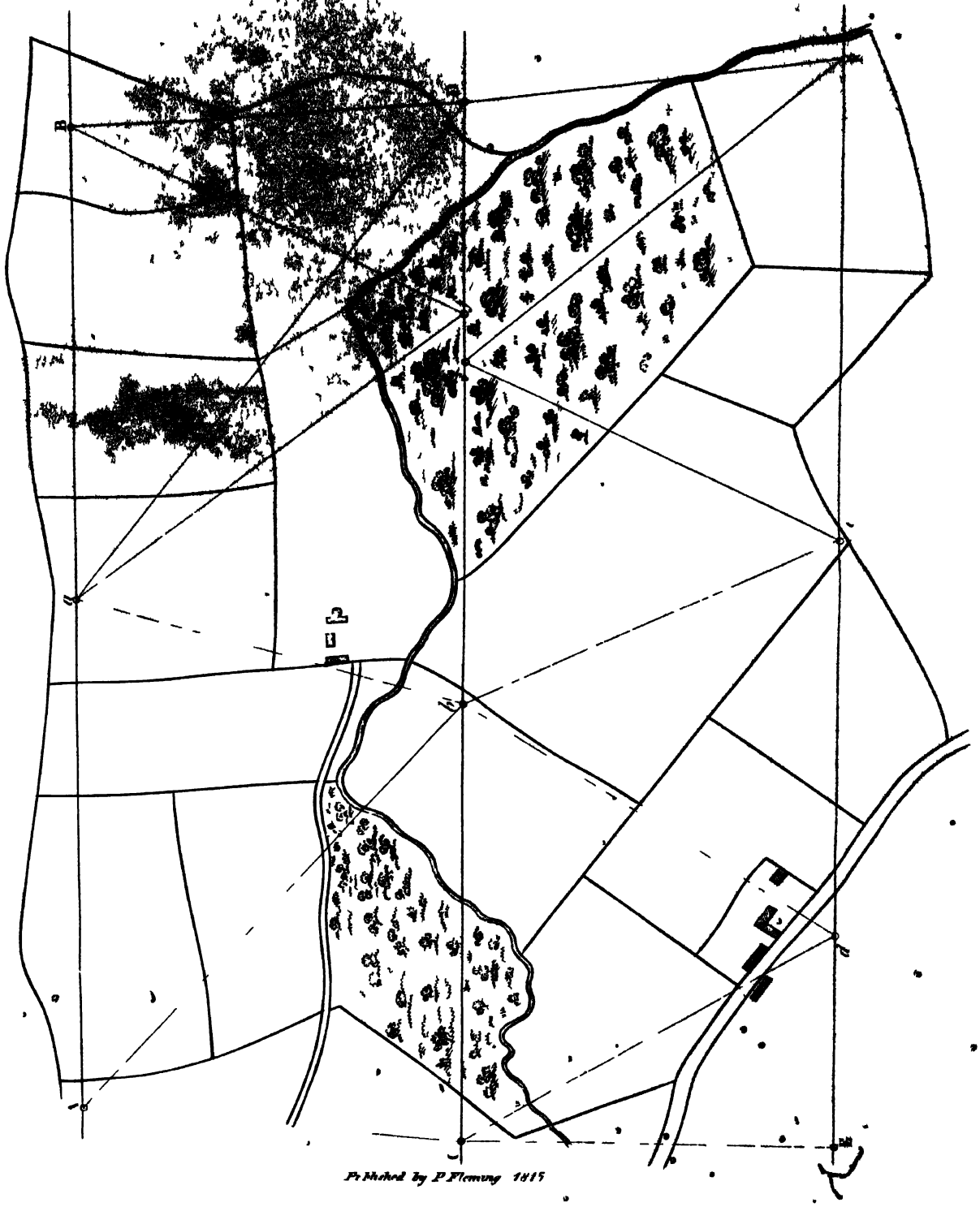
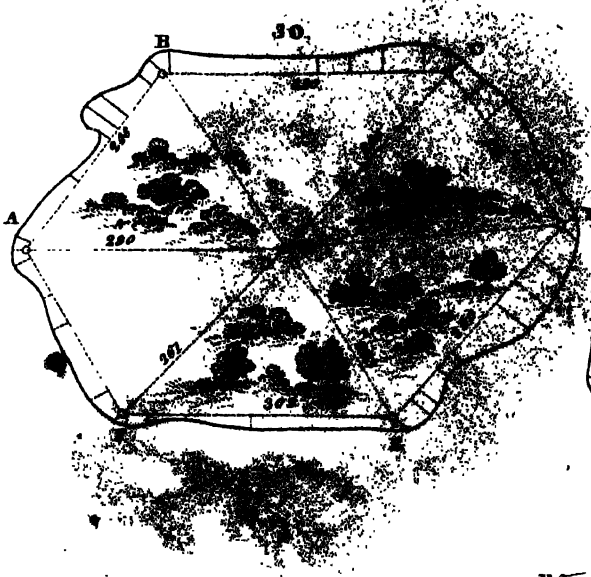
891
714
584
Published by P. Flemming, 1818

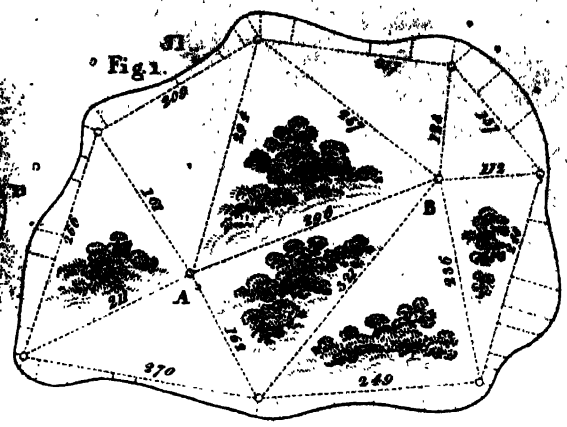


Published by P. Fleming 1815

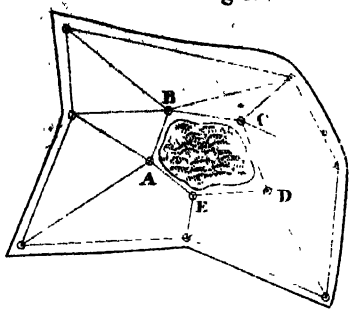
30.



31. Fig. 1.



31 Fig. 2.



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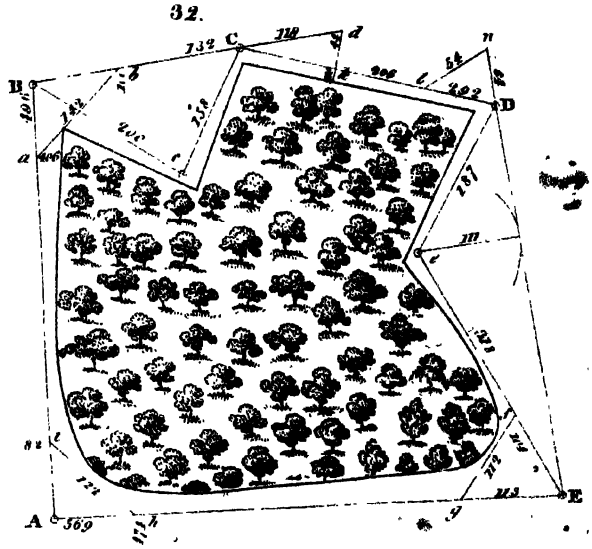

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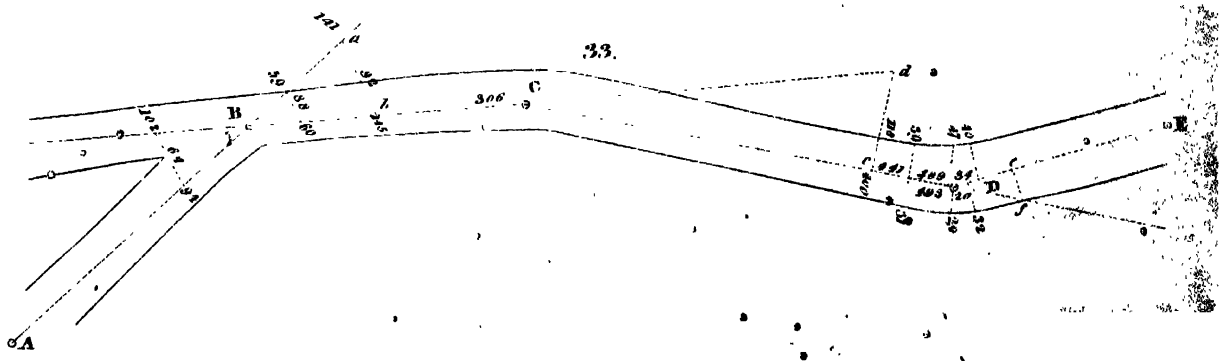
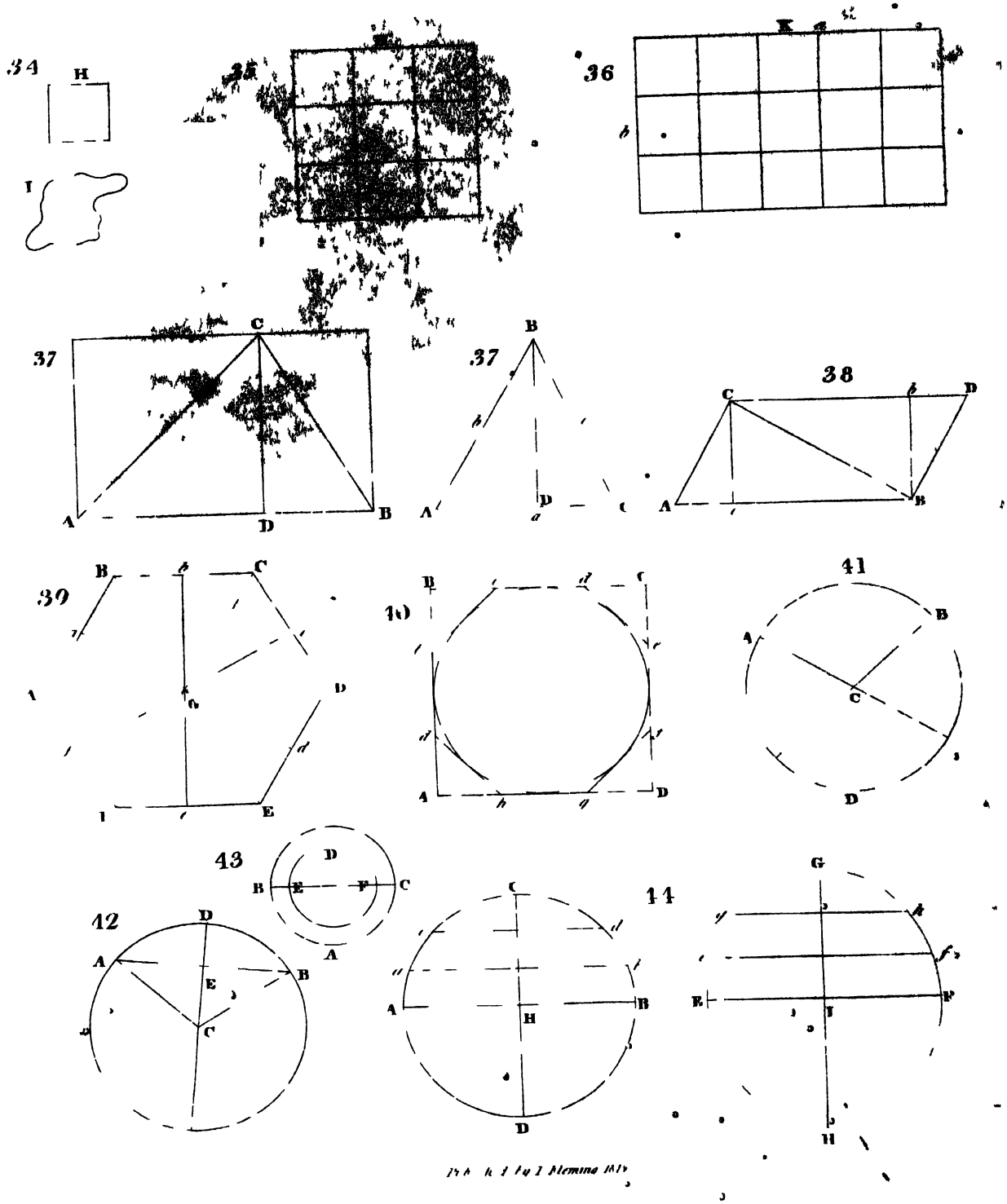


Published by P. Fleming, 1815.

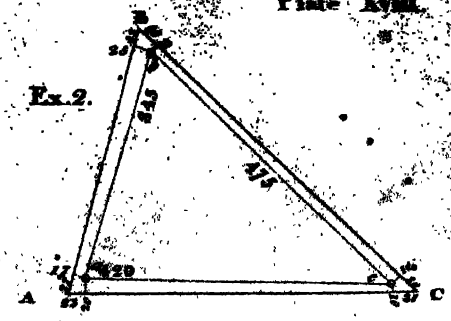
AREAS.

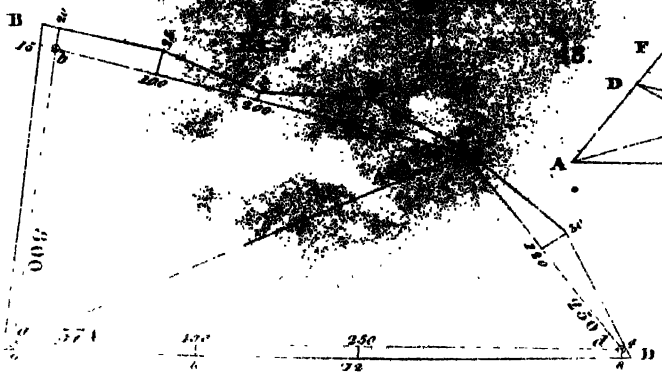
Plate XVI.



46. Ex.1.

Ex.2.





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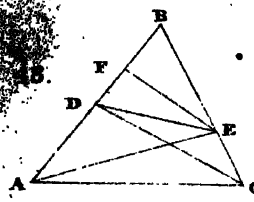
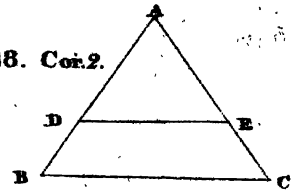


48. Cor.2.



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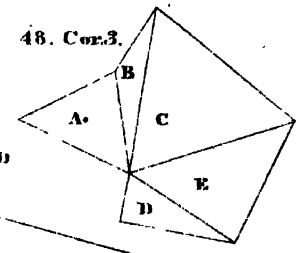
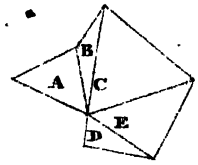
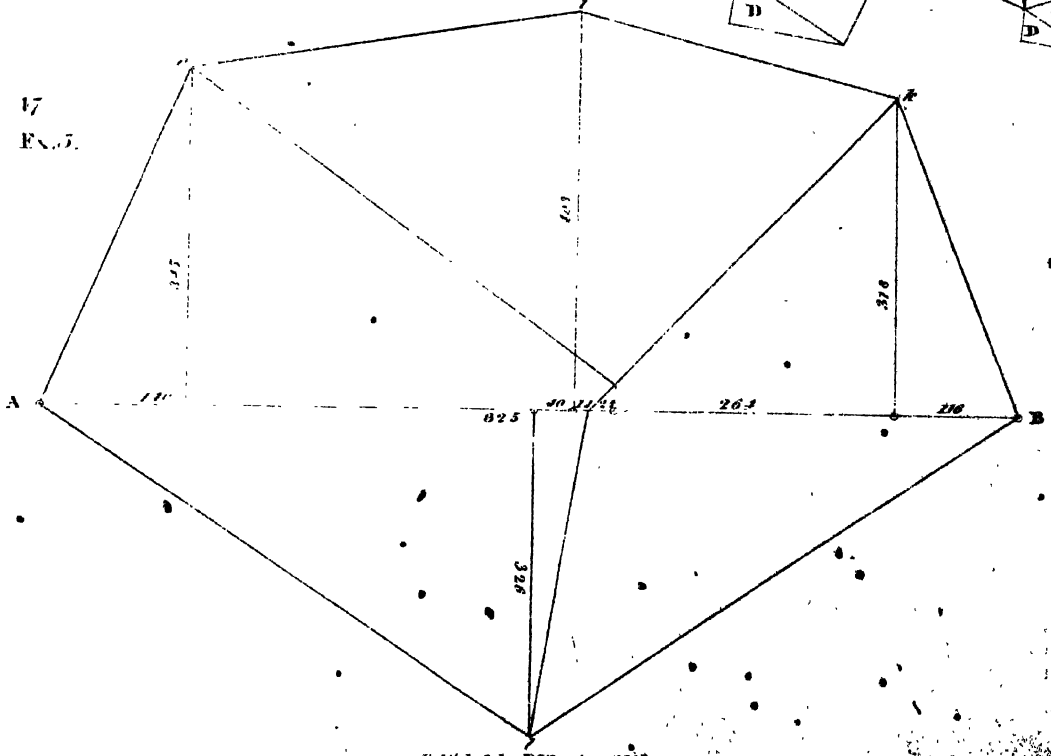




47

Ex.3.



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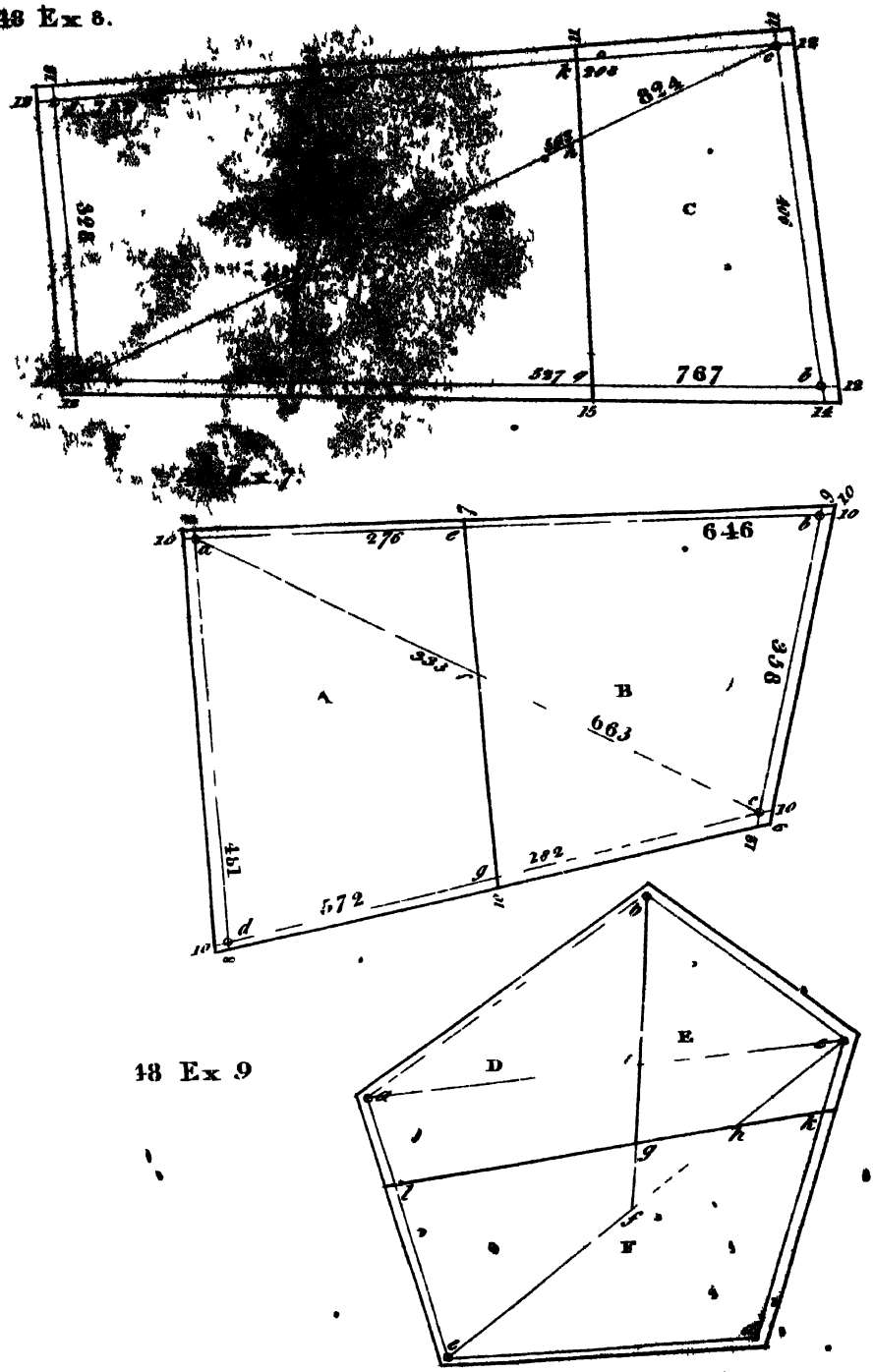
48 Ex 8.



48 Ex 9

Published by P. Flannery, 1815.